DUALITY AND ORIENTABILITY IN BORDISM THEORIES

BY DUANE O'NEILL[1]

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1. **Introduction.** A Poincaré duality theorem appears in the literature of bordism theory in several places e.g. [1], [4]. In certain $K(\pi)$-theories, i.e. classical (co)homology theories, the connection between orientability of the tangent bundle of a manifold and this duality is well known [5]. It is interesting to see how this same relationship holds in $\boldsymbol{MG}$-theories and that a simultaneous proof can be given for several different G.

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2. **Notation.** Throughout this note G_n will be one of $O(n)$, $SO(n)$, $U(n)$ or $SU(n)$. We let $\theta=\theta(G_n)$ be the disk bundle associated to the universal G_n-bundle. The Thom space, MG_n, is the total space of θ with the boundary collapsed to a point, the basepoint of MG_n. The Whitney sum of G-disk bundles induces the maps necessary to define the Thom spectrum $\boldsymbol{MG}$ and the maps giving the (co)homology products. We will denote by $(G^*(\ ))G_*(\ )$ the (co)bordism theory associated to $\boldsymbol{MG}$ as in the classical work of G. W. Whitehead [6].

Let dn be the real dimension of the fiber of θ. The inclusion of a fiber into the total space of θ can be thought of as a bundle map covering the inclusion of the basepoint into the classifying space for G_n. There is then the associated map of Thom spaces which we denote by $e_n\colon S^{dn}=D^{dn}/\partial D^{dn}\to MG_n$. If $f\colon S^qX\to MG_n$ is a map, then we denote the associated cohomology class by $(f)\in\tilde{G}^{dn}(S^qX)$. It is easy to prove using the techniques of [6] that (e_n) is the identity element of $\tilde{G}^{dn}(S^{dn})$ and that the identity element

$$e\in\tilde{G}^0(S^0)\xrightarrow{\Sigma^{dn}}(e_n)\in\tilde{G}^{dn}(S^{dn})$$

where Σ^{dn} is the iterated suspension isomorphism.

Now we consider a closed differentiable n-manifold N^n and let $\tau\colon N\to BO(2(n+k))$ be the map classifying the stable unoriented tangent bundle of N. There is the sequence

$$BSU(n+k)\to BU(n+k)\to BSO(2(n+k))\to BO(2(n+k)).$$

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