ON THE SEMICOMPLEXES OF F. BROWDER

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1. **Introduction.** In [4] Lefschetz defined quasi-complexes and proved his fixed point theorem for that class of spaces. Since then, further discussions of quasi-complexes have appeared in [1], [3] and [6]. Unfortunately, the question of whether or not the class of quasi-complexes contains the class of compact metric ANR's has never been settled. In [2] F. Browder introduced the concept of semicomplexes and showed that this class does contain the compact metric ANR's as well as admitting a local fixed point index for continuous maps. In §2 of this announcement the relationship between these two concepts is clarified by defining weak semicomplexes and showing that this class contains both the quasi-complexes and the semicomplexes. Furthermore, the Lefschetz fixed point theorem holds for weak semicomplexes and within this class the subclasses of quasi-complexes and semicomplexes are completely characterized. In §§3, 4 and 5 new results on the existence and uniqueness of semicomplexes are given.

The proofs of these and additional results will appear elsewhere. The author wishes to thank Professor Edward Fadell for suggesting several of the questions which are considered in this work.

2. **Weak semicomplexes.** The definition of a weak semicomplex is motivated by the following definition of a semicomplex which is a slight modification of that given by Browder in [2]. This definition is, however, sufficient for use in all of Browder's results and proofs.

If X is a compact Hausdorff space, $\Sigma(X)$ will denote the set of all finite covers of X by open sets. If $\alpha\in\Sigma(X)$ then N_α will stand for the nerve of α and $C(N_\alpha)$ will stand for the associated chain complex of N_α with rational coefficients. When α and β are in $\Sigma(X)$ with β refining α (i.e., $\beta>\alpha$), π_α^β: $C(N_\beta)\rightarrow C(N_\alpha)$ will denote the usual chain map induced by a vertex transformation based on set inclusion.

DEFINITION (1). A semicomplex, $S(X)=\{X, \mathcal{I}, \Omega, \alpha_0, C_\lambda\}$, is a quintuple where X is a compact Hausdorff space; $\mathcal{I}$ is a collection of finite covers of X by connected open sets which is cofinal in $\Sigma(X)$; Ω is a cofinal subset of $\Sigma(X)$; α_0 is a function from $\mathcal{I}$ into Ω such that for each $\lambda\in\mathcal{I}$, $\alpha_0(\lambda)>\lambda$; and C is a function assigning to $\lambda\in\mathcal{I}$ a family,

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