RESTRICTED REPRESENTATIONS OF CLASSICAL LIE ALGEBRAS OF TYPES A_2 AND B_2[1]

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The dimensions of the finite dimensional irreducible restricted modules for a Lie algebra of classical type have never been determined. C. W. Curtis ([**5**], [**6**]) has given sufficient, but not necessary, conditions that the dimension be given by Weyl's formula. In this paper, we give results which determine the dimensions of a certain class of finite dimensional irreducible restricted modules for a simple algebra of type A_2 or B_2 over a field of characteristic $p>3$. Counterexamples to a conjecture of N. Jacobson regarding the complete reducibility of certain modules for an algebra of classical type are given. Our method involves rather lengthy, though elementary, calculations (given in detail in [**3**]), and depends on a character formula for algebras of types A_2 or B_2 over the complex field, which is due to J. P. Antoine ([**1**], [**2**]). R. Steinberg has mentioned that the results for A_2 were obtained by Mark ([**9**]).

1. **Preliminaries.** Let $\mathfrak{L}$ be a simple Lie algebra over the complex field with Cartan subalgebra $\mathfrak{H}$. If $p>3$ is a prime number, let $\bar{\mathfrak{L}}$, $\bar{\mathfrak{H}}$ be the classical Lie algebra and corresponding Cartan subalgebra over the field $Z/(p)$ obtained from $\mathfrak{L}$ and $\mathfrak{H}$ by reducing the structure constants with respect to a Chevalley basis, modulo p (see e.g., [**10**]). The finite dimensional irreducible restricted $\bar{\mathfrak{L}}$-modules were shown by Curtis to be uniquely determined by their maximal weights [**4**]. Given a linear function $\bar{\Lambda}$ on $\bar{\mathfrak{H}}$, let Λ be the integral linear function on $\mathfrak{H}$ such that for $i=1, 2, \cdots l$, $\Lambda(h_i)$ is the integer between 0 and $p-1$ whose residue modulo p is $\bar{\Lambda}(h_i)$. (Here $h_1, \cdots, h_l$ is a basis of co-roots for $\mathfrak{H}$, corresponding to a system of fundamental roots $\alpha_1, \alpha_2, \cdots, \alpha_l$, and we have identified these elements of $\mathfrak{H}$ with their counterparts in $\bar{\mathfrak{H}}$.) Then Λ is a dominant integral function on $\mathfrak{H}$, and hence is the highest weight of a unique finite dimensional irreducible $\mathfrak{L}$-module V. Now let $\bar{V}$ be the (not necessarily restricted) $\bar{\mathfrak{L}}$-module corresponding to V (see [**5**]). Curtis showed ([**5**]) that the finite dimensional irreducible restricted $\bar{\mathfrak{L}}$-module with maximal

[1] The results presented in this paper are a part of the author's Ph.D. thesis at the University of Oregon, written under the direction of Professor C. W. Curtis.