NONLINEAR ACCRETIVE OPERATORS IN BANACH SPACES

BY FELIX E. BROWDER

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Introduction. Let X be a real Banach space, X^* its conjugate space with the pairing between w in X^* and x in X denoted by (x, w). The duality mapping J of X into 2^{X^*} is given by

$$(1) \qquad J(x) = \{w \mid w \in X^*; \|w\| = \|x\|; (x, w) = \|x\| \cdot \|w\|\},$$

for each x in X. For any Banach space X and any element x of X, $J(x)$ is a nonempty closed convex subset of the sphere of radius $\|x\|$ about zero in X^*. If X^* is strictly convex, J is a singlevalued mapping of X into X^* and is continuous from the strong topology of X to the weak* topology of X^*. J is continuous in the strong topologies if and only if the norm in X is C^1 on the complement of the origin.

DEFINITION. *If T is a (possibly) nonlinear mapping with domain $D(T)$ in X and with range $R(T)$ in X, then T is said to be accretive if for each pair x and y in $D(T)$,*

$$(2) \qquad (T(x) - T(y), J(x - y)) \geqq 0,$$

i.e. $(T(x)-T(y), w) \geqq 0$ *for all w in $J(x-y)$.*

If X^* is strictly convex, the nonlinear accretive operators from X to X coincide with the J-monotone operators studied in Browder [**3**], [**4**] and Browder-de Figueiredo [**8**]. For linear T, T is accretive if and only if $(-T)$ is dissipative in the sense of Lumer-Phillips [**10**]. Functional equations for nonlinear accretive operators have also been considered by Vainberg [**15**] and related classes of operators and problems have been studied by Hartman [**9**], Mamedov [**11**], Murakami [**13**], and Petryshyn [**14**].

It is our object in the present paper to present a number of general existence theorems for solutions of nonlinear functional equations involving nonlinear accretive operators which drastically improve earlier results in this direction as given in the papers mentioned above. The detailed proofs of these general existence theorems are given in another paper [**7**]. In the second section of the present paper, we give under somewhat sharper hypotheses a procedure of projectional or Galerkin type for computing such solutions. The convergence proof which is given in full is of special interest because it depends in an essential way upon the fact that the existence theorem for solutions has been given an independent proof. (We should also remark that the proofs of the existence theorems of [**7**] are also constructive but in a more complicated fashion.)