AN ISOMORPHISM PRINCIPLE IN GENERAL TOPOLOGY

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Introduction. To practically every topological space T of importance (including metrizable and locally compact Hausdorff spaces) one can let correspond (essentially by interchanging compact and closed sets) an "antispace" T^* which conversely determines T. If, for example, T is Hausdorff but not compact, T^* will be T_1, compact and superconnected (every open set is connected). One sacrifices the Hausdorff property but gains e.g. compactness. Furthermore the topology of T^* is weaker than that of T. This destroys the belief, generally held, that non-Hausdorff spaces are of minor or no importance. On the contrary, one could even say that they are "more elegant," since they perform the same job with a weaker topology.

Philosophically, the consequences seem to be of interest. If R denotes "time" (the real line), R^* has the same topology as R on every bounded closed interval. However R^* is compact. Time becomes unbounded but finite in the sense of compact. We have potential but no actual infinity.

A remark by J. M. Aarts (in our joint work on cocompactness) initiated this note.

Preliminaries. Let X be a set and $\{G\}$ a family of subsets G of X, closed under finite unions and arbitrary intersections. We do *not* assume the (usual) convention that X and $\emptyset$ are necessarily members of $\{G\}$. A pair $T_-=(X, \{G\})$ is called a (topological) *minusspace*, where $\{G\}$ indicates the family of closed sets of T_-. One can, of course, extend every T_- to a *topological* space T by adding X and $\emptyset$ as closed sets.

A subset S of T_- is called *squarecompact relative* to T_-, if for every family $\{C_\alpha\}$ of compact subsets C_α of T_-, for which $\{S\cap C_\alpha\}$ is centered (that is the intersection of finitely many $S\cap C_\alpha$ is nonempty), the intersection of all $S\cap C_\alpha$ is nonempty.

One can prove:

(i) The intersection of a compact and a squarecompact set is both compact and squarecompact.

(ii) The union of finitely many and the intersection of any number of squarecompact sets is squarecompact.

(iii) If in T_- every compact set is closed, then every closed set is squarecompact.