THE EQUIVALENCE OF SOME GENERAL COMBINATORIAL DECISION PROBLEMS

BY W. E. SINGLETARY[1]

Communicated by D. Scott, December 22, 1966

1. Introduction. A decision problem for a combinatorial system shall denote a pair (Φ, S) where Φ is a specified kind of decision problem (e.g. word problem, halting problem, etc.) and S is a combinatorial system. Likewise, a general combinatorial decision problem, i.e. a decision problem for a class of combinatorial systems, shall denote a pair (Φ, C), where Φ is a specified kind of decision problem and C is a general class of combinatorial systems (e.g. Turing machines, semi-Thue systems, etc.). Clearly, each general combinatorial decision problem P has a class of decision problems for combinatorial systems associated with it. We shall refer to these problems simply as the problems associated with P.

There are many papers in the literature which deal with the reduction of one general combinatorial decision problem to another. These papers fall into two general groups. The first group consists of unsolvability proofs such as [1], [8], [10], [11] and [14]. The general format of these proofs is the following: Two general combinatorial decision problems P_1 and P_2 are considered, where P_1 is known to be unsolvable. Then an effective one-one mapping Ψ of the problems p associated with P_1, into the problems associated with P_2 and a uniformly effective reduction of p to $\psi(p)$ are given. The second group consists of proofs of the existence of a problem of each r.e. degree of unsolvability associated with some general combinatorial decision problem such as [2], [3], [5], [7], [12] and [13]. The general format of these proofs is the following: Two general combinatorial decision problems P_1 and P_2 are considered, where P_1 is known to have an associated problem of each r.e. degree of unsolvability. Then an effective one-one mapping ψ of the problems p associated with P_1 into the problems associated with P_2 and uniformly effective reductions of p to $\psi(p)$ and of $\psi(p)$ to p are given.

Our aim here is to link several of these reductions together in such a way as to provide an effective proof of the equivalence of a number of general combinatorial decision problems. Furthermore, all of our reductions will conform to the second format given above and hence for each pair P_i, P_j of general combinatorial decision problems con-

[1] This research was partially sponsored by NSF Grant GP-5445.