CANONICAL FORMS OF CERTAIN VOLTERRA INTEGRAL OPERATORS AND A METHOD OF SOLVING THE COMMUTATOR EQUATIONS WHICH INVOLVE THEM[1]

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The similarity properties of Volterra operators on $L_p[0, 1]$ having reasonably smooth kernels seem to depend entirely on the behavior of the kernel as regards zeros and singularities on the diagonal $x=y$.

If T_G is a Volterra operator on $L_p[0, 1]$, then a study of its similarity properties seems to reduce to the following procedure involving the complex kernel $G(x, y)$.

(1) Classify $G(x, x)$ according to its zeros and singularities on the interval $0 \leqq x \leqq 1$.

(2) Show that T_G is similar to a unique T_p for T_p a canonical kernel of the class of which $G(x, y)$ belongs.

See [1], [2], and [4] for $G(x, y)$ of order $\alpha > 0$ i.e.

$$G(x, y) = (x - y)^{\alpha-1} H(x, y)/\Gamma(\alpha)$$

with $H(x, x) > 0$ and $H(x, y)$ having certain smoothness properties. The canonical form in this case is KJ^α for

$$K = \left[\int_0^1 [H(t, t)]^{1/\alpha} dt \right]^\alpha$$

and

$$J^\alpha f = \int_0^x ((x - y)^{\alpha-1}/\Gamma(\alpha)) f(y) dy.$$

See [5] and [6] for $G(x, y)$ of rank 1, i.e.

$$G(x, x) < 0 \quad \text{if } 0 \leqq x < x_0,$$
$$G(x, x) > 0 \quad \text{if } x_0 < x \leqq 1,$$
$$G(x_0, x_0) = 0.$$

The canonical form in this case is $kQ_{a,\nu}$ for unique real k, a, and ν satisfying $0 \leqq a$, $\nu \leqq 1$, $0 < k$

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