DE RHAM THEOREMS ON SEMIANALYTIC SETS

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Semianalytic sets are subsets of real analytic manifolds locally defined by inequalities of real analytic functions. We refer to [5] for precise definitions and properties about them. Let M be a semianalytic set in the real analytic manifold X.

We consider in this note the complexes of differential forms and currents induced on M by the smooth forms and currents of X, and relate them to the real cohomology and homology of M. The following version of de Rham's theorems holds: there is an epimorphism from the cohomology of the forms on M onto the cohomology of M, and there is a monomorphism from the homology of M into the homology of the currents on M. In general these maps are not isomorphisms, even on algebraic sets in $\boldsymbol{R}^2$. These results answer a question posed by Norguet [7].

We show in the first section that homology and cohomology classes of M can be represented by semianalytic chains and cochains. This is used in the second section to prove de Rham's theorems. In the third section an example is given in which the above maps are not isomorphisms, together with some particular remarks on the Poincaré lemma.

It is supposed throughout this note that X is paracompact and that M is closed in X and has dimension p. Then the set M^* of the regular points of M is an analytic submanifold of X and the singular set $\partial M = M - M^*$ is semianalytic in X with dimension $\dim \partial M < p$. If N is semianalytic and locally closed in X with $\dim N \leqq q$, then $bN = \overline{N} - N$ is semianalytic and closed in X with $\dim bN < q$.

Unless stated otherwise, K is a principal ideal domain. If ϕ is a family of supports on the locally compact space Y and $\mathfrak{F}$ is a sheaf of K-modules on Y, then $H_*(Y; \mathfrak{F})(H_*^\phi(Y; \mathfrak{F}))$ denotes the Borel-Moore homology of Y with coefficients in $\mathfrak{F}$ and closed supports (supports in ϕ) [1]. If $F \subset Y$ is closed, there is an exact sequence

$$\cdots \to H_q(F; K) \xrightarrow{i_{F,Y}} H_q(Y, K) \xrightarrow{j^{Y,Y-F}} H_q(Y - F; K) \xrightarrow{\partial_{Y,F}} H_{q-1}(F; K) \to \cdots.$$

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