SOME REMARKS ON THE EXTENDED DOMAIN OF FOURIER TRANSFORM

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Communicated by A. Zygmund, December 1, 1966

1. Preliminaries. We shall state in this section definitions and results concerning general integral transformations, which will be needed in the second part of the paper. The proofs of these results can be found in [1].

Let (X, μ), (Y, ν) be two σ-finite measure spaces, ϕ and ψ be measurable, positive almost everywhere functions on X and Y, such that $\int_X \phi(x)d\mu(x) = \int_Y \psi(y)d\nu(y) = 1$ and denote by ρ_X and ρ_Y the translation invariant metrics defined for measurable, finite a.e. complex valued functions on X and Y by the formulas

$$(1)\qquad \begin{aligned} \rho_X(u) &= \int_X \frac{|u(x)|}{1 + |u(x)|}\phi(x)d\mu(x), \\ \rho_Y(v) &= \int_Y \frac{|v(y)|}{1 + |v(y)|}\psi(y)d\nu(y). \end{aligned}$$

The spaces $\mathfrak{M}$ and $\mathfrak{N}$ of all measurable finite a.e. functions on X and Y respectively, when provided with metrics ρ_X and ρ_Y become complete linear metric spaces. The topologies induced by these metrics are equivalent to topologies of convergence in measure on all subsets of finite measure.

If $K(x, y)$ is a measurable, complex valued function defined on $X \times Y$, then the proper domain of the corresponding integral transformation K is defined by

$$(2)\qquad \begin{aligned} \mathfrak{D}_K = \Big\{ u \in \mathfrak{M}\colon (|K||u|)(y) \\ = \int_X |K(x, y)||u(x)|\,d\mu(x) < \infty \text{ a.e. } \nu \Big\} \end{aligned}$$

and for $u \in \mathfrak{D}_K$ the integral transformation $K\colon \mathfrak{M} \to \mathfrak{N}$ is given by

$$(3)\qquad (Ku)(y) = \int_X K(x, y)u(x)d\mu(x).$$

K is nonsingular if there exists a function $u \in \mathfrak{D}_K$ such that $u > 0$ a.e. From now on we suppose that K is nonsingular. If A is a linear metric