WIENER INTEGRAL REPRESENTATIONS FOR CERTAIN SEMIGROUPS WHICH HAVE INFINITESIMAL GENERATORS WITH MATRIX COEFFICIENTS

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1. **Introduction.** Let $\mathbf{X}=(x_t+\infty, M_t, P_x)$ be the Wiener process on $\boldsymbol{R}^n$, the real vector space of n-tuples.[2] We assume the process is separable in the sense of Doob so that if Ω is the sample space for X, then $P_x\{\omega\in\Omega \mid x_{(\cdot)}(\omega)$ is continuous on $[0, \infty)\}=1$ for all $x\in\boldsymbol{R}^n$. Let $B_1, B_2, \cdots, B_n$, and V be bounded continuous complex $N\times N$ matrix-valued functions on R^n and let $\tilde{D}(B; V)$ be the closure in $L^2(\boldsymbol{R}^n; \boldsymbol{C}^N)$ of the differential operator

$$D(B; V) = \frac{1}{2}\sum_{j=1}\left(\frac{\partial}{\partial x^j}\right)^2 + \sum_{j=1}^{n} B_j(x)\frac{\partial}{\partial x^j} + V(x)$$

with domain $C_0^\infty(\boldsymbol{R}^n; \boldsymbol{C}^N)^1$. ($C_0^\infty(\boldsymbol{R}^n, \boldsymbol{C}^N)$ denotes the space of infinitely differentiable $\boldsymbol{C}^N$-valued functions on $\boldsymbol{R}^n$ with compact support.)

It is the purpose of this announcement to state two theorems which prove the existence of an $N\times N$ matrix-valued functional $\alpha_t(B; V)$ on $\Omega\times[0, \infty)$ such that

$$\{T_t(P; V)\phi\}(x) \equiv M_x\{\alpha_t(P; V)\phi(x_t)\},$$

for $\phi\in L^2(\boldsymbol{R}^n; \boldsymbol{C}^N)$, defines a strongly continuous semigroup of operators on $L^2(\boldsymbol{R}^n; \boldsymbol{C}^N)$ with infinitesimal generator $\tilde{D}(B; V)$.

Note that if $-\tilde{D}(B; V)$ is the Schrödinger operator for a physical system, *perhaps involving internal spin*, and $\beta=1/BT$, then $T_\beta(B; V)$ is the quantum statistical matrix for the system in question. It is this fact which motivated our interest in the problem discussed in this paper. See Ginibre [9] for an interesting discussion of the application of Wiener integral representations to statistical mechanics.[3]

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[2] See [6, Chapter VII]. We shall use the notation introduced by Dynkin without further comment.

[3] The restrictions we have placed on the $\{B_j\}$ and V would only allow us to apply the Wiener integral representation theory to approximations to real situations. It should be remarked that it is not much more difficult to handle the somewhat more realistic case where $V=V_1+V_2$ and where V_1 is as above and V_2 is a real scalar-valued function which is bounded from above on $\boldsymbol{R}^n$ and continuous except for a set of capacity zero i.e., when V_2 is a repulsive potential.