CONTINUOUS STATE BRANCHING PROCESSES

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1. **Introduction.** A class of Markov processes having properties resembling those of ordinary branching processes, but with continuous instead of discrete states, was introduced by M. Jirina in [3]. Recently it has developed that these processes (assuming continuous time parameter, stationary transition probabilities, and one-dimensional state space) form precisely the class of possible limiting processes for a sequence of Galton-Watson, or simple branching, processes which have their time and space units expanding at suitable rates to infinity [5]. The purpose of this announcement is to describe the construction of the most general process of the above sort. It turns out that every such process can be obtained by a *random time change* from a process with stationary independent increments which cannot jump to the left. We will state these results precisely in §3 below, and discuss some details and examples in §4. Proofs of the main results will appear elsewhere.

2. **Definitions and examples.** The exact class of processes we will consider has been defined in [5], where some elementary properties and examples are also given. We repeat here only the essentials.

DEFINITION. A 'C.B. function' is a Markov transition function on the Borel sets of $[0, \infty)$ with $P_t(x, [0, \infty))=1$ and such that $P_t(x, E)$ is jointly measurable in t and x for each E, is nontrivial in the sense that $P_t(x, \{0\})<1$ for some $t>0$, $x>0$, and that the 'branching property'

$$P_t(x+y, \cdot) = P_t(x, \cdot) * P_t(y, \cdot) \tag{1}$$

is satisfied for all $t, x, y \geqq 0$, where $*$ denotes convolution.

DEFINITION. A 'C.B. process' is a Markov process on $[0, \infty)$ with right-continuous paths whose transition probabilities are given by a C.B. function.

In [5] it is shown that a C.B. function must be stochastically continuous and map the space of continuous functions on $[0, \infty]$ into itself; it is a consequence that every C.B. process is automatically strong Markov.

We will work with the spatial Laplace transforms of C.B. functions, which can be written in the form

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