# THE LOCAL RING OF THE GENUS THREE MODULUS SPACE AT KLEIN'S 168 SURFACE

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1. **Introduction.** In [6], as a synthesis of earlier papers of mine, I give, in the form of a set of prescriptions for local coordinates, a description of $M^g$, the space of conformal equivalence classes of compact Riemann surfaces of genus $g$, as a complex space. Particular interest attaches to those points (surface classes) of $M^g$ representing surfaces admitting conformal self-maps (automorphisms) because, outside of certain cases for $g=1, 2, 3$ (over and above the elliptic and hyperelliptic involutions for $g=1, 2$), these points are singular (nonuniformizable) points in the structure. In particular for $g\geqq 2$, where one needs $3g-3$ complex parameters to describe $M^g$ near a generic point, one needs $3g-3+\rho$, $\rho>0$, near one of the points in question.[2] According to Prescription III ([6, p. 17]) the problem reduces to finding an irreducible basis for the homogeneous nonconstant, polynomial invariants of a finite group of linear transformations in $3g-3$ variables, namely, the hermitian adjoint of the group induced on the quadratic differentials of a representative surface of the point in question by the conformal automorphism group of that surface.

For a finite nonabelian linear group, while there is an algorithm for computing some basis for the invariants (cf. Prescription III), there is notoriously no known algorithm for computing an *irreducible* basis, i.e., for discarding the superfluous ones. Accordingly I felt it of interest to illustrate the whole phenomenon by a nontrivial example. To anyone who has worked on the subject the one that immediately comes to mind is Klein's surface of genus three admitting as automorphism group a representation of the simple group of order 168 ([1], [3]). This example commends itself in that it is of "maximum complexity" in the sense that it admits its full quota according to Hurwitz [2] of 84 $(g-1)=168$, $(g=3)$ automorphisms (on the subject of such surfaces see the interesting papers [4], [5]). It develops, *vide infra*, that eleven invariants, i.e., $11=3\cdot 3-3+5=6+5$, so $\rho=5$, are needed to generate the local ring of $M^3$ at Klein's surface class.[2]

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[2] With $\rho$ relations on "syzygies," of course.