MULTIPLICATIVE FIBRE MAPS

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In this note we shall outline a result concerning the cohomology of a multiplicative fibre map. To fix our notation we shall assume that

$$F \xrightarrow{i} E \xrightarrow{\pi} B$$

is a Serre fibre space such that

(1) F, E, B are H-spaces (homotopy associative) and $F \to E$, $E \to B$ are H-maps.

(2) B is simply connected.

(3) $H^*(B;\ Z_p)$ is a polynomial algebra, where Z_p denotes the integers modulo p, p a prime.

(4) $H_*(B;\ Z_p)$ is a commutative algebra.

The result that we shall establish is

THEOREM. *If $H^*(E;\ Z_p)$ and $H^*(B;Z_p)$ are of finite type and p is an odd prime, then*

$$H^*(F;Z_p) \cong \operatorname{Tor}_{H^*(B;\ Z_p)}(Z_p,\ H^*(E;Z_p))$$

as an algebra over Z_p. (A similar result holds over the rationals Q.)

The result for $p=2$ is more complicated to state and is treated in Theorem 3.

In fact, as we shall see, we can compute the indicated torsion product simply from a knowledge of the cohomology map

$$\pi^*\colon H^*(B;Z_p) \to H^*(E;Z_p).$$

Results and techniques similar to these have been used in [8] to compute the Z_p-cohomology of stable two stage Postnikov systems.

This announcement serves as an introduction to the joint work of J. C. Moore and the author that will appear elsewhere.

1. Algebra. Throughout this section k will denote a fixed field and $\otimes$ will mean $\otimes_k$. We shall assume that the reader is familiar with the material covered in the homological algebra section of [1]. All mod ules are assumed of finite type. All algebras will be assumed graded augmented and connected.

DEFINITION. If Γ is a Hopf algebra over k, an ideal $I \subset \Gamma$ is called

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