DELPHIC SEMIGROUPS

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A delphic semigroup shall be a topological commutative semigroup which is Hausdorff and possesses a neutral element and satisfies the three conditions (A–C) below. In formulating these we require some terminology: a triangular array is a system $u(i, j)$ $(i=1, 2, \cdots; j=1, 2, \cdots, i)$ of elements of the semigroup; the ith marginal product is the element

$$u(i, 1)u(i, 2) \cdots u(i, i);$$

an array is said to converge to an element u when the ith marginal product converges to u; an element is said to be infinitely divisible when it possesses a kth root for each $k \geqq 2$.

(A) There exists a continuous homomorphism Δ from the semigroup into the additive semigroup of nonnegative reals, such that $\Delta(u)=0$ if and only if u is the neutral element.

(B) The set $\{u' : u' | u\}$ of factors of any given element u is compact.

(C) If a triangular array converges to u, and if the array satisfies the condition $\Delta(u(i, j)) \to 0$ as $i \to \infty$ uniformly for $1 \leqq j \leqq i$, then u is infinitely divisible.

As a nontrivial example we mention here only the multiplicative semigroup of positive renewal sequences; for the complete details of this and other examples, as well as for the proofs of the following theorems, reference should be made to [1] and [2].

The first property of a delphic semigroup is that every infinitely divisible element u can be represented as a limit as in (C). Next, it can be shown that the elements of such a semigroup can be partitioned into three exhaustive and mutually exclusive classes; the elements in the first class are indecomposable; those in the second class are decomposable but possess an indecomposable factor; those in the third class are infinitely divisible and possess no indecomposable factor. Finally it can be shown that an arbitrary element of such a semigroup possesses at least one representation in the form

$$u = v(1)v(2) \cdots w,$$

where the v's are indecomposable and w is infinitely divisible and possesses no indecomposable factor. There are not more than countably many v's; there may be none.