# ORLICZ SPACES OF FINITELY ADDITIVE SET FUNCTIONS, LINEAR OPERATORS, AND MARTINGALES[1]

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The purpose of this note is to announce some properties and applications of Orlicz spaces of finitely additive set functions, the $V^\Phi$ spaces. The $V^\Phi$ spaces are natural generalizations of the $V^p$ spaces (Bochner [2] and Leader [6]).

1. **The $V^\Phi(\mathfrak{X})$ spaces.** Throughout this note $\Omega$ is a point set, $\Sigma$ a field of subsets of $\Omega$, $\mu$ a finitely additive extended real valued nonnegative set function defined on $\Sigma$; and $\Sigma_0 \subset \Sigma$ is the ring of sets of finite $\mu$-measure. A partition $\pi$ is a finite disjoint collection $\{E_n\} \subset \Sigma_0$. The partitions are partially ordered by defining $\pi_1 \leqq \pi_2$ whenever each $E_n \in \pi_1$ is a union of members of $\pi_2$. $\mathfrak{X}$ and $\mathfrak{Y}$ are Banach (or $B$-) spaces with conjugate spaces $\mathfrak{X}^*$ and $\mathfrak{Y}^*$ respectively. $\Phi$ is a (nontrivial) Young's function with complementary function $\Psi$.

DEFINITION. $V^\Phi(\Omega, \Sigma, \mu, \mathfrak{X}) = (V^\Phi(\mathfrak{X}))$ consists of all finitely additive $\mu$-continuous $\mathfrak{X}$-valued set functions $F$ on $\Sigma_0$ such that for some $k > 0$,

$$I_\Phi(F/k) = \sup_\pi \sum_\pi \Phi\left(\frac{\|F(E_n)\|}{k\mu(E_n)}\right)\mu(E_n) \leqq 1,$$

where the supremum is taken over all partitions $\pi = \{E_n\}$ and the convention $0/0 = 0$ is observed.

$V^\Phi(\mathfrak{X})$ becomes a $B$-space under each of the equivalent norms

$$N_\Phi(F) = \inf\{k > 0 \colon I_\Phi(F/k) \leqq 1\}$$

or

$$\|F\|_\Phi = \sup\left\{\sup_\pi \sum_\pi \frac{\|F(E_v)\| \, \|G(E_v)\|}{\mu(E_v)} : G \in V^\Psi(\mathfrak{X}^*), N_\Psi(G) \leqq 1\right\}.$$

Using the integration procedure of [4, Chap. III], one can define the (possibly incomplete) Orlicz spaces $L^\Phi(\Omega, \Sigma, \mu, \mathfrak{X})(=L^\Phi(\mathfrak{X}))$ of totally $\mu$-measurable $\mathfrak{X}$ valued functions $f$ satisfying $\int_\Omega \Phi(\|f\|/k)d\mu \leqq 1$ for some $k > 0$. $L^\Phi(\mathfrak{X})$ becomes a normed linear space under either of the two equivalent norms $N_\Phi(f) = \inf\{k > 0 \colon \int_\Omega \Phi(\|f\|/k)d\mu \leqq 1\}$ or, if

[1] The results announced here are contained in the author's doctoral thesis written under the guidance of Professor M. M. Rao at Carnegie Institute of Technology.