DEGREES OF IRREDUCIBLE MODULAR CHARACTERS OF BLOCKS WITH CYCLIC DEFECT GROUPS

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Let G be a finite group of order $g'p^a$, p a prime, $(g', p)=1$. Let D be a cyclic group of order p^d, $d \geqq 1$, and let B be a block of G with D as a defect group. We obtain the following theorem:

THEOREM. *If ϕ is an irreducible modular character of B (with respect to p), then the degree of ϕ must be congruent* (mod p^{a-d+1}) *to one of the following values*: $-eN, -(e-1)N, \cdots, -2N, -N, +N, 2N, \cdots, (e-1)N, eN$, *where* $N \not\equiv 0$ (mod p^{a-d+1}), *and e (defined below) divides $p-1$. In particular, the degree of ϕ is not divisible by p^{a-d+1}. If B is the principal block, then* $N \equiv 1$ (mod p).

The theorem is already known for $d=a=1$. We prove the theorem here by considering the tree associated with the block B. Brauer [1] describes this tree in discussing blocks with $d=1$. Dade [2] generalizes Brauer's results to arbitrary d, and his results enable us to associate the same kind of tree with the block B. The tree turns out to have a certain number e (defined below) of edges. The question of whether or not every tree with e edges is associated with some block of a group remains open. We now describe the tree (for details see Dade [2]).

Let C_0 be the centralizer and N_0 the normalizer of D in G. Let b_0 be some block of C_0 such that $b_0^G=B$, and let E be the subgroup of N_0 fixing b_0 under conjugation. Denote by e the index $(E: C_0)$. Then e divides $p-1$. The irreducible characters of B can be divided into $e+1$ classes, of which e classes consist of one nonexceptional character each, and one class consists of the exceptional characters. (If there are no exceptional characters, then $e=p^{a-d+1}-1=p-1$, and there are $e+1$ nonexceptional characters. In this case we choose one of them and treat it below as the exceptional class.) There are exactly e irreducible modular characters of B, and each one occurs as a constituent of just those irreducible characters in exactly two of the classes.

We can represent this information with a graph. Let the vertices of the graph correspond to the classes of irreducible characters of B,

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