SMOOTH AND PIECEWISE LINEAR SURGERY

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We shall be concerned here with developing some new techniques of surgery on a map in both the smooth and p.l. (piecewise linear) categories. In [11] these tools are applied to the problem of deforming a homotopy equivalence between two piecewise linear manifolds until it is a piecewise linear homeomorphism, and the Hauptvermutung (which claims that topologically homeomorphic p.l. manifolds are piecewise linearly homeomorphic) is answered affirmatively for a large class of manifolds.[2]

The general problem of surgery on a map may be described as follows: Consider a finite CW pair $(X, \partial X)$ satisfying Poincaré duality in dimension n (cf. [12]), a k-dimensional bundle $E_x \to X$, and a map f: $(W, \partial W) \to (V, \partial V)$ from a compact $(n+k)$-manifold $(W, \partial W)$ to a pair of spaces $(V, \partial V)$ containing $(E_x, E_x|\partial X)$ as an open subpair. When can f be deformed (as a map of pairs) until it is transverse regular on $X \subset V$ in such a way that $(M, \partial M) = (f^{-1}(X), f^{-1}(\partial X)) \subset (W, \partial W)$ is an n-submanifold with normal bundle $E_m = f^*E_x$ and f: $(M, \partial M) \to (X, \partial X)$ is a homotopy equivalence covered by the bundle map f: $E_m \to E_x$? If this can be done, we say that f may be deformed until it is *h-regular* on X. If $f|\partial W$ is already h-regular on ∂X, when can f be deformed rel ∂W (i.e. keeping $f|\partial W$ fixed) until it is h-regular on X? For example, the pioneering work of Browder [1] and Novikov [10] dealt, in the smooth category, with the case where $W = S^{n+k}$, $V = TE_x =$ the Thom space of E_x, and $f =$ a map of degree one. Wall's generalization in [12] of the Browder-Novikov results uses $(W, \partial W) = (D^{n+k}, S^{n+k-1})$, $(V, \partial V) = (T(E_x), T(E_x|\partial X))$, and a map of degree one. Our main interest will be when f is a homotopy equivalence and the codimension k is different from two. Furthermore, the surgery results are true in both the smooth and p.l. categories, and the theorems are stated simultaneously for both cases. Thus all maps, manifolds, bundles, hypotheses, conclusions, etc. should be interpreted as all smooth or all p.l. unless the category is explicitly pinned down. For an exposition of the p.l. category, p.l. microbundles, and p.l. transverse regularity see [13]. Recall that by the Kister-Mazur Theorem every p.l. microbundle contains a unique *p.l. bundle*—a

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