# SUBGROUPS OF FINITE GROUPS[1]

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**1. Introduction.** Let $G$ be a finite group. What can we say about $G$ if we are given some information about the subgroups of $G$? That is, what does the local structure of $G$ tell us about the global structure of $G$? In this paper we will describe some answers to this question and some remaining unsolved cases. We will concentrate on the following special case:

*Problem* 1. Given a particular subgroup $H$ of $G$, what can we say about $G$?

Throughout this paper, $G$ will denote an arbitrary finite group, and all groups considered will be finite.

**2. Centralizers of involutions.** Suppose $\tau$ is an element of $G$. Let $C(\tau)$ be the centralizer of $\tau$, i.e., the set of elements of $G$ that commute with $\tau$. Many of the answers to Problem 1 in recent years have concerned the case in which $H=C(\tau)$ and $\tau$ is an *involution*, that is, an element of order two. Why are elements of order two different from elements of odd prime order? The reason is a paradox: involutions occur in both the hardest groups and the easiest groups with which we have to deal.

In many problems about finite groups, the hardest cases are the nonsolvable groups and, in particular, the simple groups. (We say that $G$ is simple if it is *not* abelian and if it has no normal subgroups other than itself and 1, the identity subgroup. Thus we exclude the cyclic groups of prime order.) The celebrated theorem of Feit and Thompson [7] asserts that if $G$ is not solvable, then the order, or number of elements, of $G$ is even. Thus $G$ has a nonidentity Sylow 2-subgroup, which, in turn, must contain an involution.

THEOREM 1 (FEIT-THOMPSON). *If $G$ is not solvable, then $G$ contains an involution.*

Thus, if $G$ is not solvable, involutions are available. But this does not guarantee that we can handle them; here is where the "easiest" groups come in. Suppose we want to study the local properties of $G$ in some abstract, general way. Nothing could be more local than an

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