AUTOMORPHISMS OF OPERATOR ALGEBRAS

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1. Introduction. We describe certain results concerning the structure of the group $\alpha(\mathfrak{A})$ of automorphisms of a C^*-algebra $\mathfrak{A}$. They will appear, together with their proofs, in Communications in Mathematical Physics.

All mappings of $\mathfrak{A}$ we consider preserve the * structure. With ϕ a faithful representation of $\mathfrak{A}$ on the Hilbert space $\mathcal{H}$, we say that an automorphism β of $\phi(\mathfrak{A})$ is weakly-inner when there is a unitary operator U in the strong-operator closure $\phi(\mathfrak{A})^-$ of $\phi(\mathfrak{A})$ such that $\beta(A)=UAU^*$ for all A in $\phi(\mathfrak{A})$. We denote by $\iota_\phi(\mathfrak{A})$ the subgroup of $\alpha(\mathfrak{A})$ consisting of those α for which $\phi\alpha\phi^{-1}$ is weakly-inner, by $\epsilon_\phi(\mathfrak{A})$ those α such that $\phi\alpha\phi^{-1}$ extends to an automorphism of $\phi(\mathfrak{A})^-$, by $\sigma_\phi(\mathfrak{A})$ those α such that there is some unitary operator U on $\mathcal{H}$ for which $\phi\alpha\phi^{-1}(A)=UAU^{-1}$ when A lies in $\phi(\mathfrak{A})$ and by $\pi(\mathfrak{A})$ the intersection of all $\iota_\phi(\mathfrak{A})$. We refer to the elements of $\sigma_\phi(\mathfrak{A})$, $\epsilon_\phi(\mathfrak{A})$ and $\pi(\mathfrak{A})$ as the *spatial*, *extendable*, *permanently weakly* (for brevity, $\pi-$) *inner* automorphisms, respectively, (of $\phi(\mathfrak{A})$ in the first two instances and $\mathfrak{A}$ in the last). We denote by $\iota_0(\mathfrak{A})$ the group of inner automorphisms of $\mathfrak{A}$.

The group $\alpha(\mathfrak{A})$ consists of operators on the Banach space $\mathfrak{A}$ each of which is isometric so that $\alpha(\mathfrak{A})$ acquires a norm (or metric) topology as a subset of the bounded operators on $\mathfrak{A}$. We consider $\alpha(\mathfrak{A})$ and the various subgroups defined as provided with this topology. We denote by $\gamma(\mathfrak{A})$ the connected component of the identity element ι of $\alpha(\mathfrak{A})$.

2. **The automorphism group.** It has been proved recently [**3**], [**4**], [**5**] that each derivation δ of a C^*-algebra $\mathfrak{A}$ acting on the Hilbert space $\mathcal{H}$ is weakly inner—that is, there is a B in $\mathfrak{A}^-$ such that $\delta(A)=BA-AB$ for all A in $\mathfrak{A}$. Combining this result with those of Nagumo-Yosida and some series computations, we have:

LEMMA 1. *Each norm-continuous one-parameter subgroup of* $\alpha(\mathfrak{A})$ *lies in* $\pi(\mathfrak{A})$.

Spectral theory and von Neumann algebra considerations yield:

LEMMA 2. *If* α *is an inner automorphism of a von Neumann algebra*

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