LINEARLY CONTINUOUS FUNCTIONS OF FINITE AREA[1]

BY CASPER GOFFMAN

Communicated by A. Zygmund, June 24, 1966

1. **Definitions and preliminaries.** We shall consider real functions of n variables with compact support.

Let $x=(x_1, \cdots, x_n)$ be a cartesian coordinate system and let $\bar{x}_i=(x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$ be a point in $n-1$ space, so that $x=(x_i, \bar{x}_i)$. A function f is said to be of type BVT if there are equivalent functions f_i, $i=1, \cdots, n$, such that, for almost all $\bar{x}_i$, the variation $V_i(f, \bar{x}_i)$ of f as a function of x_i is finite, and V_i is a summable function of $\bar{x}_i$. The functions of type BVT are those for which the partial derivatives are measures, [9], and those for which the area is finite, [1], [4]. f is said to be ACT if it is BVT and the equivalent f_i, $i=1, \cdots, n$, may be chosen to be absolutely continuous for almost all $\bar{x}_i$. These are the functions whose partial derivatives are functions, [3], [11], and for which the area is given by the classical formula [4].

A function f is said to be essentially linearly continuous if, for every $i=1, \cdots, n$, there is an equivalent f_i which is continuous as a function of x_i for almost all $\bar{x}_i$. f is said to be linearly continuous if there is a g equivalent to f such that, for every $i=1, \cdots, n$, g is continuous in x_i for almost all $\bar{x}_i$. It is known, [5], that every essentially linearly continuous function which is of type BVT is linearly continuous, and that in the case of functions of two variables these are the ones for which the area is equal to the Hausdorff 2 dimensional measure of the graph. Linearly continuous functions which are of type BVT will be called of type L.

Let f be of type BVT, let $(\mu_1, \cdots, \mu_n)$ be its gradient measure, and let m be Lebesgue measure. The total variation α_f of the vector valued measure $(m, \mu_1, \cdots, \mu_n)$ is the area measure, [2], [8].

Functions of types BVT, ACT, and L may be discontinuous everywhere. However, they may be considered to be the respective analogues of the functions of bounded variation, the absolutely continuous functions, and the continuous functions of bounded variation in the one variable case.

[1] Supported by National Science Foundation Grant No. GP 03515. A detailed paper will appear in Acta Mathematica.