A NONABELIAN TWO-DIMENSIONAL COHOMOLOGY FOR ASSOCIATIVE ALGEBRAS[1]

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In a previous paper [4], a nonabelian cohomology theory for associative algebras was proposed for dimensions 0 and 1. The purpose of this paper is to extend this theory to dimension 2. The methods used are closely analogous to those employed in the corresponding theory for groups (cf. [1],[2]). Throughout this paper all algebras will be associative algebras over some fixed commutative ring Λ with identity. The term *homomorphism* without qualification will mean homomorphism of Λ-algebras.

1. **One-cochains and one-cocycles.** We consider algebras B and M, and homomorphisms $\rho: B\to M$, $\Phi: M\to\mathfrak{M}(B)$, where $\mathfrak{M}(B)$ denotes the algebra of bimultiplications of B (cf. [4],[5]). The system $\mathfrak{B}=(B, M; \rho, \Phi)$ is said to define the structure of an *M-crossed module* on B if the following conditions are satisfied:

(i) the image of Φ is *permutable* on B;

(ii) $\rho(\Phi(m)b)=m\rho(b)$, $\rho(b\Phi(m))=\rho(b)m$, for all $b\in B$, $m\in M$;

(iii) the composite $\Phi\rho: B\to\mathfrak{M}(B)$ maps each element of B onto the inner bimultiplication which it defines.

We recall that a subset S of $\mathfrak{M}(B)$ is *permutable* on B if $(\xi b)\eta=\xi(b\eta)$, for all $\xi, \eta\in S$, $b\in B$ (cf. [4],[5]). The M-crossed module B is an M-bimodule under the action defined by Φ, and the condition (ii) shows that ρ is a homomorphism of M-bimodules. Moreover $\rho(B)$ is an ideal in M and the quotient algebra $L=M/\rho B$ has a canonical map $\Psi: L\to\mathfrak{M}(Z)$, deduced from Φ, where $Z=\operatorname{Ker}(\rho)$ is called the *center* of the crossed module $\mathfrak{B}$. M and L will be called the *operator* and *outer operator* algebras of $\mathfrak{B}$ respectively.

Given $\mathfrak{B}=(B, M; \rho, \Phi)$, we define a 1-*cochain* from an algebra A to $\mathfrak{B}$ to be a pair (p, ϕ) of maps $p: A\to B$, $\phi: A\to M$ and denote the set of these 1-cochains by $\mathfrak{C}^1(A, \mathfrak{B})$. $\mathfrak{Z}^1(A, \mathfrak{B})$ is the subset consisting of the 1-*cocycles*, namely those pairs (p, ϕ) for which

$$\begin{aligned}&\phi \text{ is a homomorphism;}\\ (1)\quad &p \text{ is a homomorphism of } \Lambda\text{-modules;}\\ &p(a_1a_2) = p(a_1)p(a_2) + p(a_1)\phi(a_2) + \phi(a_1)p(a_2), \qquad a_1, a_2 \in A.\end{aligned}$$

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