BLOCK BUNDLES

BY C. P. ROURKE AND B. J. SANDERSON

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Block bundles play the same role in the piecewise linear category, as vector bundles in the differential category. This is an announcement of results supporting this claim, details will appear elsewhere. All maps and spaces are assumed p.l.

A *q-block bundle* ξ^q/K consists of a total space $E(\xi)$ and a locally finite simplicial complex $K \subset E(\xi)$, satisfying

(i) For each n-simplex $\sigma_i \in K \exists$ an $n+q$-ball $\beta_i \subset E$ such that (β_i, σ_i) is an unknotted ball pair. β_i is the *block* over σ_i.

(ii) $E(\xi)$ is the union of the balls β_i.

(iii) If $\sigma_i \cap \sigma_j = \sigma_k$ (or $\emptyset$) then $\beta_i \cap \beta_j = \beta_k$ (or $\emptyset$).

Block bundles ξ^q, η^q/K are *isomorphic* if $\exists$ a homeomorphism $h: E(\xi) \to E(\eta)$, which preserves the block structure and such that $h|K = \text{identity}$.

Now let K' be a subdivision of K, and suppose given ξ/K, then ξ'/K' is a *subdivision* of ξ/K if $E(\xi) = E(\xi')$, and the union of blocks of ξ' lying over a simplex $\sigma_i \in K$ is the block β_i of ξ. The converse of subdivision is *amalgamation.*

Let $I_q(K)$ be the set of isomorphism classes of q-block bundles over K, and let $|K|$ denote the polyhedron underlying K.

THEOREM 1. *Suppose $|K| = |L|$ and J a common subdivision. Then the operation of subdivision over J, followed by amalgamation induces a bijection*

$$I_q(K) \to I_q(L)$$

which is independent of the choice of J.

Now let X be a polyhedron (a space with a related family of locally finite triangulations) and suppose $|K| = |L| = X$. ξ/K is *equivalent* to η/L if they have isomorphic subdivisions. Denote the set of equivalence classes by $I_q(X)$ (isomorphic to $I_q(K)$ any $|K| = X$, in a natural way by Theorem 1).

Induced bundles (and Whitney sums) can be defined and $I_q(\ )$ becomes a contravariant homotopy functor on the category of polyhedra. In fact, using an analogue of the Grassmannian we have

THEOREM 2. *There is a locally finite simplicial complex $B(\boldsymbol{PL})_q$ and a block bundle $\gamma^q/B(\boldsymbol{PL})_q$ such that*