# FIBER COBORDISM AND THE INDEX OF A FAMILY OF ELLIPTIC DIFFERENTIAL OPERATORS

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There is a relation between Hirzebruch's differentiable Riemann-Roch theorem [12] of Grothendieck type, and the index formula of Atiyah-Singer [1] for elliptic operators. We shall try to establish such a relation by using a suitable generalization of Thom [14], Conner-Floyd [4] cobordism theory. As a consequence,[1] the analytical index [6] of an elliptic family is computed and shown to be a topological invariant [8] modulo torsion. The author expresses his deep gratitude to H. Cartan, A. Grothendieck, L. Illusie, R. Palais and R. Thom for their indispensable help and kind encouragement.

1. **Notation.** All manifolds $M$ are smooth compact orientable and without boundary. $\Lambda = \boldsymbol{Z}[1/2]$ denotes the subring of the rational numbers $\boldsymbol{Q}$ whose denominators are a power of 2; and

$$k^*_\Lambda(X) = KU^*(X) \otimes z\Lambda, \qquad f^!: \ {}^*_\Lambda(X) \to K^*_\Lambda(Y),$$

denote the natural homomorphism in $K$-theory induced by a map $f: Y \to X$. If $\pi: \eta \to X$ is an oriented $n$-dimensional vector bundle over a CW-complex $X$, we shall denote by $\hat{\pi}: \hat{\eta} \to X$, $q: \hat{\eta} \to \dot{\eta}$, $j: X \to \hat{\eta}$ the associate $n$-dimensional sphere bundle over $X$, its projection onto the Thom space $\dot{\eta}$ and the embedding induced by the zero section of $\eta$.

For a finite CW-complex $B$, $C_B$ is the category of all fiber bundles $p: X \to B$, over $B$, with fiber an orientable manifold $M$ and structural group a subgroup of all orientation preserving diffeomorphisms of $M$. We shall denote by

$$\pi: T_\natural(X) \to X$$

the cotangent bundle along the fiber of $p: X \to B$. A map $f: X \to Y$ in $C_B$ is supposed to be fiber preserving and $C^\infty$ along each fiber. If $\xi$ is a complex vector bundle over $X$, $J^k_\natural(X) \to X$, $k \in \boldsymbol{Z}_+$, will denote the $k$-jet bundle [11] along the fiber, characterized by the restriction of $J^k_\natural(X)$ on a fiber $M$ is the $k$-jet bundle of the restriction of $\xi$ on $M$. In a way similar to that of Palais [9] the *Sobolev chain of* $\xi$ is well defined: $H^s_\natural(\xi) \to B$, $s \in \boldsymbol{R}$, as Hilbertian vector bundles over $B$ whose fibers are the Sobolev chain $H^s(\xi | M)$. Now if $\eta$ is another complex

[1] This gives an answer to a problem of Chern (Bull. Amer. Math. Soc. **72** (1966), 167–219), in fact we can obtain thus a Riemann-Roch theorem of Grothendieck for the Chern character of the direct image of the sheaf of holomorphic sections of a locally trivial holomorphic vector bundle.