# RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

## NORMAL OPERATORS, LINEAR LIFTINGS AND THE WIENER COMPACTIFICATION[1]

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Recently A. S. Galbraith communicated to the authors the conjecture that normal operators (Nakai-Sario [4]) are linear liftings (Tulcea [7]). In the present Research Announcement we shall show that the conjecture is correct: conditions (1)–(5) of [4] imply conditions (I)–(V) of [7].

Galbraith's conjecture also led us to a generalization of normal operators where we make use of Wiener's compactification of a Riemann surface (cf. Constantinescu-Cornea [2]). The results have applications to Ahlfors' [1] conjecture on extreme normal operators.

We wish to express our sincere gratitude to Dr. Galbraith for stimulating this research.

1. **Normal operators on Wiener's boundary.** Consider a finite union $R=\bigcup_{j=1}^{n} R_j$ of disjoint hyperbolic Riemann surfaces $R_j$ with Wiener harmonic boundaries $\Gamma_j$ (Constantinescu-Cornea [2]). Decompose $\Gamma_j$ into two disjoint compact sets $\alpha_j$ and $\beta_j$, the case $\beta_j=\emptyset$ (void) not excluded. Set $\Gamma=\bigcup_1^n \Gamma_j$, $\alpha=\bigcup_1^n \alpha_j$ and $\beta=\bigcup_1^n \beta_j$.

We are interested in mappings of the space $C(\alpha)$ of real-valued continuous functions on $\alpha$ into the space $H(R)$ of harmonic functions on $R$. An operator $L$ from $C(\alpha)$ into $H(R)$ is, by definition, *normal if* (L.1) *$L$ is linear,* (L.2) *$f\geqq 0$ implies $Lf\geqq 0$,* (L.3) $Lf|\alpha=f$, (L.4) $L1=1$, *and* (L.5) $\int_\gamma {}^*dLf=0$ *along a dividing cycle $\gamma$ on $R$ homologous to $\alpha$.*

Clearly $Lf\in HB(R)$, the space of bounded functions in $H(R)$. If $R$ is a bordered Riemann surface with compact border $\bar{\alpha}$, and if $\alpha$ lies on $\bar{\alpha}$, then $L$ is normal in the original sense [4], as stated more precisely in §4 below.

In the case $\beta=\emptyset$, an operator $L$ satisfying (L.1) and (L.2) gives

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