A COUNTEREXAMPLE ON RELATIVE REGULAR NEIGHBORHOODS

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Hudson and Zeeman defined the concept of a relative regular neighborhood in [1], and gave an existence theorem and two uniqueness theorems; the purpose of this note is to show that the uniqueness theorems are false. A corrected version of these theorems has been announced by L. S. Husch and will appear later. For 3-manifolds, however, the corrected version is equivalent to the original.

For general terminology and definitions, see Zeeman [2]. Suppose K, L are subcomplexes of some complex J. We say that K is *link collapsible* on L if $lk(A, \mathrm{Cl}(K-L))$ is collapsible for any simplex A of $\mathrm{Cl}(K-L)\cap L$. If X and Y are compact polyhedra in a polyhedral manifold M, we say that X is link collapsible on Y if there is a triangulation K, L of X, Y such that K is link collapsible on L. For example, it is easy to see that a manifold is always link collapsible on any subpolyhedron of its boundary. Let X, Y, N be compact polyhedra in M. We say that N is a *regular neighborhood of* X mod Y *in* M if

(1) N is an m-manifold ($m = \dim M$),

(2) N is a topological neighborhood of $X - Y$ in M and

$$N \cap Y = N \cap Y = \mathrm{Cl}(X - Y) \cap Y,$$

(3) N collapses to $\mathrm{Cl}(X - Y)$.

The uniqueness theorems given by Hudson and Zeeman say, among other things, that *any two regular neighborhoods of* X mod Y *in* M *are homeomorphic keeping* $\mathrm{Cl}(X - Y)$ *fixed*, provided X is link collapsible on Y.

Let (B^3, B^1) be a knotted 3, 1 ball pair in E^3 and let $B^4 = a * B^3$ and $B^2 = a * B^1$ where $a = (0, 0, 0, 1) \in E^4$ and $*$ denotes join. The 4, 2 ball pair (B^4, B^2) is locally knotted at the vertex a and hence is knotted. However it is easy to see that B^2 is unknotted in E^4. Let $h: E^4 \to E^4$ be an onto piecewise linear homeomorphism such that $h(B^2) = \Delta$ is a 2-simplex. B^4 collapses cone-wise to B^2, so that $h(B^4)$ collapses to $h(B^2) = \Delta$. Also $\dot{\Delta} \subset h(\dot{B}^4)$ and $\mathring{\Delta} \subset h(\mathring{B}^4)$, so that $h(B^4)$ is a regular neighborhood of Δ mod $\dot{\Delta}$ in E^4. Let Σ be the 2-fold suspension of Δ in E^4; then Σ is a regular neighborhood of Δ mod $\dot{\Delta}$ in E^4.

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