ADAPTED INTEGRAL REPRESENTATIONS BY MEASURES ON CHOQUET BOUNDARIES

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1. **Introduction.** In recent years, the following abstract Cauchy-Weil Theorem has been established by different methods [8], [7], [4]:

Let X be a relatively compact domain in $\mathbf{C}^n$ or more generally in a separable analytic space $(Y, \mathfrak{O})$. Let H be the algebra of continuous complex valued functions on $\overline{X}$ which are holomorphic in X. Then there exists a family $(\mu_x)_{x\in X}$ of complex measures on the Šilov boundary S of $\overline{X}$ with respect to H such that:

(i) $\mu_x(f)=f(x)$ for all $f\in H$, $x\in X$.

(ii) $x\to\mu_x(g)$ is holomorphic on X for all continuous complex valued functions g on S.

H. Bauer [1] has raised the problem whether it is possible to concentrate the measures μ_x on the Choquet boundary $X_e(H)$ of $\overline{X}$ with respect to H. $X_e(H)$ is defined (for all separating vector spaces H of scalar valued functions on a compact space $\overline{X}$) as the set of all $x\in\overline{X}$ such that the unit point measure ϵ_x at x is the only positive Radon measure μ of total mass 1 on $\overline{X}$ satisfying $\mu(h)=h(x)$ for all $h\in H$ [6]. If $\overline{X}$ is metrizable and H is a Banach algebra containing the constants $X_e(H)$ coincides with the minimal boundary of H as defined in [3]. One can prove that for an *algebra* H the following statements are equivalent: (a) $x\in X_e(H)$; (b) if $\mu(h)=h(x)$ for all $h\in H$, then $\mu(\{x\})\neq 0$ for all complex measures μ on $\overline{X}$; (c) ϵ_x is orthogonal to all complex measures μ on $\overline{X}$ with $\mu(h)=0$ for all $h\in H$.

The purpose of the present paper is to give a solution of Bauer's problem by treating even the following more abstract situation:

Let X be a relatively compact open subset of a Hausdorff space, $\mathcal{C}(\overline{X}, K)$ the algebra of all continuous K-valued functions on $\overline{X}$ ($K=\mathbf{C}$ or $K=\mathbf{R}$) and H a separating linear subspace of $\mathcal{C}(\overline{X}, K)$ with $1\in H$. Suppose E to be a vector space of K-valued functions on X which contains the restrictions $\mathrm{res}_X\, h$ of all $h\in H$ to X—under which conditions is there a family $(\nu_x)_{x\in X}$ of (K-valued) Radon measures concentrated on the Choquet boundary $X_e=X_e(H)$ such that:

(i) (ν_x) is a family of H-representing measures, i.e. $\nu_x(h)=h(x)$ for all $h\in H$, $x\in X$;

(ii) (ν_x) is E-adapted, i.e. $x\to\nu_x(g)$ is in E whenever $g\in\mathcal{C}(\overline{X}_e, K)$?

Important suggestions for our solution were given by L. Bungart [4].