ON REGULAR NEIGHBORHOODS OF SPHERES

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Consider the following two conjectures:

$C(n)$: (The combinatorial Schoenflies conjecture.) A combinatorial $(n-1)$-sphere on a combinatorial n-sphere decomposes the latter into two combinatorial n-cells.

$D(n)$: Let W^n be an orientable combinatorial manifold without boundary and let M^{n-1} be a closed orientable combinatorial manifold imbedded piecewise linearly in W^n. Let U be a regular neighborhood of M^{n-1} in W^n. Then there exists a piecewise linear homeomorphism $h: M^{n-1}\times[-1;1]\to U$ such that

$$h(x,0)=x, \tag{1}$$

$$h \text{ is onto.} \tag{2}$$

It is easily seen that $D(n)$ implies $C(n)$ for all $n\neq 4$ by using the Hauptvermutung for combinatorial cells and spheres [**10**]. In [**8**], Noguchi shows that $C(1), C(2), \cdots, C(n)$ imply $D(n+1)$. By using the fact that a compact component of the boundary of a combinatorial manifold is combinatorially collared [**9**], [**11**], it is easily shown that $C(n)$ implies $D(n+1)$. However it is possible to prove a weaker version of $D(n+1)$ without the use of $C(n)$ for the special case when W, M are spheres.

THEOREM. *Let $\sum^n$ $(n\neq 4)$ be a combinatorial sphere embedded piecewise linearly in the combinatorial sphere S^{n+1}. Let U be a regular neighborhood of $\sum^n$ in S^{n+1}. Then there exists a piecewise linear homeomorphism $h: \sum^n\times[-1;1]\to S^{n+1}$ such that $h(\sum^n\times[-1;1])=U$.*

PROOF. (For definitions of terms used see [**11**].) Since $C(i)$, $i=1,2,3$, is valid [**1**], [**6**], it follows from the remarks above that the theorem is true for $n<4$. Suppose $n>4$.

Since $\sum^n$ is a deformation retract of U, the ith integral homology groups of $\sum^n$ and U are isomorphic for all i. It follows then from Alexander duality and the unicoherence of the sphere that the closure of $S^{n+1}-U$, $\mathrm{Cl}(S^{n+1}-U)$, is the union of two connected closed sets,

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