THE K THEORY OF THE PROJECTIVE UNITARY GROUPS

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Let $U=U(p^r)$ be the unitary group on complex p^r space, p an odd prime. Let $S^1 \subset U$ be the set of matrices λI where λ is a complex number with $|\lambda|=1$ and I is the identity matrix. Then S^1 is the center of U and $PU(p^r)=PU=U/S^1$.

We determine the complex K^* groups for the spaces PU by first determining the mod qK^* groups of these spaces [2] then using the mod p Bockstein spectral sequence to obtain the p torsion. $K^*[PU(p^r)]$ and $H^*[PU(p^r)]$ have no q torsion for $q\neq p$ and the mod p Bockstein spectral sequences for these two groups are isomorphic; thus,

THEOREM 5.5. *$H^*[PU(p^r), Z]$ and $K^*[PU(p^r)]$ are isomorphic as abelian groups.*

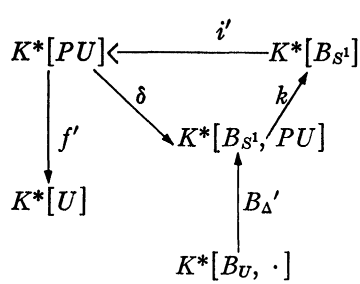
The details of these proofs will be published elsewhere. The outline follows:

Let B_{S^1} and $B_U=B_U(p^r)$ be the classifying spaces of the indicated groups. There are the following maps

$$U \xrightarrow{f} PU \xrightarrow{i} B_{S^1} \xrightarrow{B_\Delta} B_U.$$

Either i or B_Δ may be considered fibrations. We use the following diagram

(1)



Let ρ_q: $K^*[\ , Z]\to K^*[\ , Z_q]$ be the reduction [2] and β_K: $K[\ , Z_q]\to K[\ , Z]$ the Bockstein. $\exists \sigma_1, \sigma_2, \cdots, \sigma_{p^r} \in K^*[B_U, \cdot] \ni K^*[B_U]=Z[[\sigma_1, \cdots, \sigma_{p^r}]], H^*[B_U]=Z[\bar{\sigma}_1, \cdots, \bar{\sigma}_{p^r}], K^*[U]=E[s\sigma_1, \cdots, s\sigma_{p^r}]$, $K^*[B_{S^1}]=Z[[y]]$, $H^*[B_{S^1}]=Z[\bar{y}]$. ρ_p is onto for these groups and it will be convenient to use x for $\rho_p(x)$ when possible. $kB_\Delta'(\sigma_i)=C_{p^r,i}y^i$ and $B_\Delta^*(\bar{\sigma}_i)=C_{p^r,i}\bar{y}^i$.