COBORDISM OF GROUP ACTIONS

BY ARTHUR WASSERMAN[1]

Communicated by R. Palais, May 9, 1966

Let G be a compact Lie group and M a compact G manifold without boundary, i.e. a C^∞ manifold with a differentiable action of G on M. M^n is said to be G-cobordant to zero $M \sim_G 0$ if there exists a compact G manifold Q^{n+1} with $\partial Q = M$. Note that in this case M_G (the fixed point set of M) $= \partial Q_G$. M_G and Q_G are both disjoint unions of closed submanifolds (of varying dimension) of M, Q respectively. Let $\nu(M_G, M)$ denote the normal bundle of M_G in M; $\nu(M_G, M) \to M_G$ is a G-vector bundle in the sense of [5]. A partial converse to the statement $\nu(M_G, M) = \partial \nu(Q_G, Q)$ is given by

PROPOSITION 1 ([2, p. 10]). *If $\nu(M_G, M)$ is cobordant to zero as a G-vector bundle, i.e. if there exists a manifold W and a G-vector bundle $E \to W$ with $\partial W = M_G$, $E | \partial W = \nu(M_G, M)$ then M is G-cobordant to a manifold M' with $M'_G = \varnothing$.*

PROOF. Form the manifold $M \times I \cup_f E(1)$ where $E(1)$ denotes the unit disc bundle in E and

$$f: E(1) \mid \partial W = \nu(M_G, M) \xrightarrow[\text{fix}]{\exp} M \times 1.$$

Then note that, after smoothing,

$$\begin{aligned} \partial(M \times I \cup_f E(1)) &= M \times 0 \cup (M \times 1 - f(E(1) \mid \partial W) \cup \partial E(1)) \\ &= M \times 0 \cup M'. \end{aligned}$$

Hence, one may view the G-cobordism class of $\nu(M_G, M)$ as a first obstruction to finding a cobordism $M \sim_G 0$. Higher obstructions are formulated in terms of a spectral sequence. For simplicity we deal only with the unoriented case.

Let V be an orthogonal representation of G and let V^n denote the n-fold direct sum of V with itself and $S(V)$ the unit sphere in V. Consider the category of manifolds $\mathcal{G}(V)$ where M is in $\mathcal{G}(V)$ iff M can be imbedded in $S(V^n)$ for some n. One can then define the cobordism groups $\mathfrak{N}_n(V) = \mathfrak{N}_n(\mathcal{G}(V))$ of n dimensional G-manifolds in $\mathcal{G}(V)$ (see [5]). It was shown in [5] that if G is finite or abelian then $\mathfrak{N}_n(V) \approx \pi_1^{V^{2n+3}}(T_k(V^{2n+3} \oplus R), \infty)$ where $\pi_1^{V^{2n+3}}(T_k(V^{2n+3} \oplus R), \infty)$ de-

[1] This research was supported in part by the U. S. Army Office of Research, (Durham).