A FOURIER SERIES METHOD FOR MEROMORPHIC AND ENTIRE FUNCTIONS

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We associate with a function f, meromorphic in the finite complex plane, the Fourier expansion

$$\log |f(re^{i\theta})| = \sum c_k(r)e^{ik\theta}.$$

The growth of f, measured by the Nevanlinna characteristic $T(r, f)$, and the growth of the coefficients $c_k(r)$ are closely related, even in the case of functions of infinite order. The $c_k(r)$ can be expressed in terms of the local behaviour of f near the origin, as well as the distribution of the zeros and poles of f. Thus, results on the zero and pole distribution of meromorphic functions of a given rate of growth are obtained that involve the angular distribution as well as the radial distribution. In particular, entire functions with prescribed zeros are constructed without the use of canonical products. Also, it is proved that each meromorphic function of a reasonably general rate of growth is the quotient of two entire functions of the same rate of growth. Some of the results in this note were obtained for a special case in [2].

DEFINITION. *A growth function $\lambda(r)$ is a function defined for $0 \leqq r < \infty$ that is positive, nondecreasing, and continuous.*

DEFINITION. *A meromorphic function f is said to be of finite λ-type if there exist constants A and B such that $T(r, f) \leqq A\lambda(Br)$.*

REMARK. *An entire function f is of finite λ-type if and only if there exist constants A and B such that $|f(z)| \leqq \exp(A\lambda(B|z|))$.*

In case $\lambda(r) = r^\rho$, $\rho > 0$, then the functions of finite λ-type are precisely the functions of growth at most order ρ, finite type. Our considerations include functions $f(z)$ that grow like $\exp(\exp(\exp(|z|)))$ for example.

We consider sequences $Z = \{z_n\}$ of complex numbers, z_n distinct from 0, such that $|z_n| \to \infty$ as $n \to \infty$.

DEFINITION. *We say that Z is of finite λ-density if there exist constants A and B such that $N(r, Z) \leqq A\lambda(Br)$, where*

$$N(r, Z) = \sum_{|z_n| \leq r} \log\left(\frac{r}{|z_n|}\right).$$

DEFINITION. *We say that Z is λ-balanced if there exist constants A and B such that for $k = 1, 2, 3, \cdots$*