A SETTING FOR GLOBAL ANALYSIS[1]

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Introduction. The primary aim of this report is to present a broad outline for a coherent geometric theory of certain aspects of nonlinear functional analysis. Its setting requires the calculus in topological vector spaces, differential geometry of infinite dimensional manifolds, and the algebraic and differential topology of function spaces. For the most part the developments are of quite recent origin, and at present the theory is in a fluid state (its growth depending strongly on its concrete applications). The beginnings of the subject may be traced to the work of Fréchet, Gâteaux, and Volterra; we refer to the text [73] of P. Lévy for an exposition of some early applications (especially in the calculus of variations and integrable differential systems)—and ask pardon for not presenting any historical perspective in the present survey.

About ten years ago it was formally recognized [29] that many of the function spaces which arise in global geometric mathematics possess a natural infinite dimensional differentiable manifold structure. That was not a great surprise; for

(1) Many of the most interesting manifolds of differential geometry are well known to have representations as function spaces of rigid maps. (E.g., Riemannian manifolds arise as the configuration spaces of dynamical systems, their cotangent bundles are interpreted as phase spaces, and their Riemannian metrics in terms of kinetic energy.)

(2) Much of the language of the classical treatment of the calculus of variations—and the penetrating viewpoint and methods of M. Morse—is that of a function space differential geometry. (E.g., the Euler-Lagrange operator of a variational problem has an interpretation as a gradient vector field, whose trajectories are lines of steepest descent.)

(3) Certain eigenvalue problems in integral and differential equations have interpretations in terms of Lagrange's method of multipliers, involving differential geometric ideas in infinite dimensions (e.g., focal point theory, and geometric consequences of the inverse

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