

TWO THEOREMS IN GEOMETRIC MEASURE THEORY

BY HERBERT FEDERER¹

Communicated by J. Wermer, February 8, 1966

The following two propositions give some new information about the structure of differentiable maps. We use the symbols R^m and H^s to designate m dimensional Euclidean space and s dimensional Hausdorff measure, respectively.

THEOREM 1. *If $m > r \geq 0$ and $k \geq 1$ are integers, Y is a normed real vectorspace, $f: R^m \rightarrow Y$ is k times continuously differentiable, and*

$$S = R^m \cap \{x: \dim \operatorname{im} Df(x) \leq r\},$$

then

$$H^{r+(m-r)/k}[f(S)] = 0.$$

THEOREM 2. *If $f: R^m \rightarrow R^n$ is Lipschitzian, r is an integer, $0 \leq r \leq m$, and*

$$T = R^n \cap \{y: H^{m-r}(f^{-1}\{y\}) > 0\},$$

then H^r almost all of T can be covered by a countable family of r dimensional submanifolds of class 1 of R^n .

The first theorem optimally sharpens the results of [4], where the history of the problem is discussed; its proof uses a refinement of the key lemma in [3], which dealt with the case $r=0$. The second theorem is related to the coarea formulae obtained in [2] and [1]. Proofs of both theorems will appear in the author's book *Geometric measure theory*.

REFERENCES

1. J. E. Brothers, *Integral geometry in homogeneous spaces*, Trans. Amer. Math. Soc. (to appear).
2. Herbert Federer, *Curvature measures*, Trans. Amer. Math. Soc., **93** (1959), 418-491.
3. A. P. Morse, *The behavior of a function on its critical set*, Ann. of Math. **40** (1939), 62-70.
4. Arthur Sard, *Hausdorff measure of critical images on Banach manifolds*, Amer. J. Math. **87** (1965), 158-174.

BROWN UNIVERSITY

¹ This work was supported in part by a research grant from the National Science Foundation.