NOTE ON STRUCTURAL STABILITY

BY J. L. ARRAUT[1]

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Introduction. In this note we prove that if X is a structural stable vector field on a compact manifold M of dimension $m \geqq 2$, then any real valued function defined on M which is invariant under X i.e., which is constant along orbits of X, must be constant. Of course some assumptions of differentiability are made. From this follows in particular that structural stable vector fields do not admit global first integrals. Here we use the ϵ-definition of structural stability, for the non ϵ-definition (see [1]) it is still a problem.

1. We will denote by M a connected compact manifold of dimension $m \geqq 2$, with a differentiable structure of class large enough for our purpose. Let $\mathfrak{F}_k(M)$ be the set of all real valued functions of class C^k, $k \geqq 1$, defined on M and $\mathfrak{X}_k(M)$ the space of all C^k vector fields on M with the C^k-topology. Finally let $\Sigma_k(M)$ be the set of structural stable vector fields contained in $\mathfrak{X}_k(M)$. (See [1].)

Let $X \in \mathfrak{X}_k(M)$. A function $f \in \mathfrak{F}_k(M)$ is called *invariant* under X if it is constant along orbits of X. It is equivalent to say that $Xf = 0$, where Xf denotes differentiation of f with respect to the vector field X. An invariant function f under X is called a *global first integral* of X if it is nonconstant on any open subset of M.

LEMMA 1. *If $f \in \mathfrak{F}_k(M)$ $k \geqq 1$, then the set Γ of critical values of f is closed in R.*

Let $X \in \mathfrak{X}_k(M)$ $k \geqq 1$ and $f \in \mathfrak{F}_{k+1}(M)$ a nonconstant function invariant under X. We assume that f has at least one regular value and from Lemma 1 it follows then, that there is an open set of them. The inverse image $f^{-1}(a)$ of every regular value a of f is a compact manifold N_a of dimension $(m-1)$ properly imbedded in M. Since f is invariant under X every N_a is invariant under X, i.e., N_a is a union of orbits of X. Let $[a_1, a_2]$ be a closed interval made up of regular values of f, we are going to define a special kind of perturbation of X associated to the interval $[a_1, a_2]$.

Fix a positive definite Riemannian metric g on M of differentiability class large enough. Let grad f be the vector field on M defined by the equation

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