CONCORDANCE AND ISOTOPY OF PL EMBEDDINGS

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Concordance and isotopy give two equivalence relations between embeddings of manifolds in manifolds. The results announced in this paper show that, for PL embeddings of a compact manifold with co-dimension ≥ 3, these two relations are the same.

One effect of this is that unknotting theorems may be deduced immediately from embedding theorems. Such an unknotting theorem is given here (Theorem 4).

DEFINITIONS. Let M and Q be PL manifolds of dimensions m and q respectively. Let I denote the closed unit interval of the real line. A *concordance* of M in Q is a PL embedding $h: M \times I \rightarrow Q \times I$ such that $h(M \times 0) \subset (Q \times 0)$ and $h(M \times 1) \subset Q \times 1$.

An isotopy of M in Q is a concordance which is level-preserving, that is to say, $h(M \times t) \subset Q \times t$ for all t in I. $h_t: M \rightarrow Q$ is used to denote the embedding defined by $h(x, t) = (h_t x, t)$ for all x in M.

This notation is also used with $t = 0$ or 1, when h is any concordance.

If $X \subset M$, a concordance (or isotopy) h of M in Q is *fixed on* X if $h(x, t) = (h_0 x, t)$ for all (x, t) in $X \times I$.

Now suppose that M and Q have boundaries ∂M and ∂Q respectively, possibly empty.

An embedding $i: M \rightarrow Q$ is *allowable* if $i^{-1} \partial Q$ is an $(m-1)$-submanifold of ∂M, possibly empty or the whole of ∂M.

A concordance (or isotopy) h of M in Q is allowable if $h^{-1}(Q \times 0) = M \times 0$, $h^{-1}(Q \times 1) = M \times 1$, h_0 is an allowable embedding, and $h^{-1}(\partial Q \times I) = (h_0^{-1} \partial Q) \times I$.

Two embeddings f and g of M in Q are *concordant* if there is a concordance h of M in Q with $h_0 = f$ and $h_1 = g$.

Similarly we talk of two embeddings being isotopic, allowably concordant, concordant keeping a subset of M fixed, etc.

An *ambient isotopy* of Q is an isotopy of Q in itself which is a PL homomorphism and with h_0 equal to the identity.

Two embeddings f and g of M in Q are *ambient isotopic* if there is an ambient isotopy h of Q such that $h_1 f = g$.

By the isotopy extension theorem [1] and [2], the conditions allowably isotopic and ambient isotopic are equivalent for embeddings of compact manifolds with codemension ≥ 3.

THEOREM 1. *Let $h: M \times I \rightarrow Q \times I$ be an allowable concordance. If M*