HILBERT SPACE IS HOMEOMORPHIC TO THE COUNTABLE INFINITE PRODUCT OF LINES[1]

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1. Introduction. In this paper, Hilbert space, denoted by l_2, is understood to be the space of all sequences (x_i) such that $\sum_{i=1}^{\infty} x_i^2 < \infty$ with $d((x_i), (x_i')) = (\sum_{i=1}^{\infty} (x_i - x_i')^2)^{1/2}$. We let the countable infinite product of lines be regarded as $s = \prod_{i=1}^{\infty} I_i^0$ where, for each $i > 0$, I_i^0 denotes the open interval $(0, 1)$.

Let the symbol "$\sim$" mean "is homeomorphic to." We shall prove

THEOREM I. $l_2 \sim s$.

As a consequence of this theorem it is possible to investigate topological properties of l_2 as topological properties of s. In turn s is a "natural" subset of the Hilbert cube (the countable infinite product of closed intervals) which facilitates the study of s.

In 1928 in [**5**, pp. 94–96] Fréchet raised the general question as to which linear topological spaces were homeomorphic to each other. Specifically he asked whether l_2 (called Ω) was homeomorphic to s (called E_ω).

In 1932 in [**2**, p. 233], Banach stated that Mazur had shown that s was not homeomorphic to l_2. Subsequently it was understood that the question was still open.

The topological classification of complete linear metric spaces initiated by Fréchet has been the subject of considerable research activity with noteworthy contributions by Bessaga, Kadec, Klee and Pełczyński among others. See the bibliography in [**3**]. Particular attention has been given to Fréchet spaces: locally convex complete linear metric spaces. With Theorem I of this paper and recent profound results of Kadec and of Bessaga and Pełczyński, the topological classification of separable infinite-dimensional Fréchet spaces is now complete. All such spaces are homeomorphic to each other.

The results leading to this theorem are the following. In a paper to be published in Dokl. Akad. Nauk SSSR, Kadec gives a proof of the theorem "All separable infinite-dimensional Banach Spaces are homeomorphic." Earlier in [**4**] and in [**3**, Theorem 9.2], Bessaga and Pełczyński have shown "Under the conjecture that all separable in-

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