ON A CHARACTERIZATION OF TYPE I C^*-ALGEBRAS[1]

BY SHÔICHIRÔ SAKAI

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1. **Introduction.** Recently, establishing a conjecture of Calkin [1], the author [7] showed the following result: Let $\mathfrak{H}$ be a separable Hilbert space, $B(\mathfrak{H})$ the C^*-algebra of all bounded operators on $\mathfrak{H}$, $C(\mathfrak{H})$ the C^*-algebra of all compact operators on $\mathfrak{H}$, then the quotient algebra $B(\mathfrak{H})/C(\mathfrak{H})$ has a type III-factor $*$-representation. The discussions which are used in the proof of this result are applicable to more general situations. In the present paper, by using those discussions and the result of Glimm [3], we shall give a characterization of type I C^*-algebras without the assumption of separability as follows:

MAIN THEOREM. *Let A be a C^*-algebra. Then the following conditions are equivalent.*

(1) *A is a* GCR *algebra,*

(2) *A is of type* I,

(3) *A has no type* III*-factor $*$-representation.*

2. **Theorems.** First of all we shall state a generalization of the result which are crucial in the proof of Calkin's conjecture.

THEOREM 1. *Let A be a C^*-algebra with unit I, B a C^*-sub algebra containing I of A and M a type* III*-factor on a separable Hibert space. Suppose that there is a linear mapping P of A into M satisfying the following conditions*:

(1) $P(x^*)=P(x)^*$ *for* $x\in A$,

(2) $P(h)\geqq 0$ *for* $h(\geqq 0)\in A$,

(3) $P(axb)=P(a)P(x)P(b)$ *for* $a, b\in B$ *and* $x\in A$,

(4) *$P(B)$ is σ-weakly dense in M. Then, A has a type* III*-factor $*$-representation.*

The proof of this theorem is similar to the one in [7]. Here we shall sketch the proof. Let Ω be the set of all linear mappings Q of A into M satisfying the conditions (1), (2), (3) and $Q(a)=P(a)$ for $a\in B$.

Let $\mathfrak{L}(A, M)$ be the Banach space of all bounded linear mappings of A into M. Then it is the dual of a Banach space $A\otimes_\gamma M_*$, where M_* is the associated space (namely, the dual of $M_*=M$) and γ is the greatest cross norm.

LEMMA 1. *Ω is a $\sigma(\mathfrak{L}(A, M), A\otimes_\gamma M_*)$-compact convex subset of $\mathfrak{L}(A, M)$ and each $Q\in\Omega$ satisfies $Q(x^*x)\geqq Q(x)^*Q(x)$ for $x\in A$.*

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