SOLVABILITY OF THE FIRST COUSIN PROBLEM AND VANISHING OF HIGHER COHOMOLOGY GROUPS FOR DOMAINS WHICH ARE NOT DOMAINS OF HOLOMORPHY. II

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This work is a continuation of [2]. In [2] we studied the cohomology groups $H^q(X\backslash A, \mathcal{O})$ where $A(\subset X)$ is a closed generalized polydisc. Here we consider the general case where A is the closure of a domain of holomorphy. This general case was treated in [1] for $q=1$, but the present method (for $q \geqq 1$) is entirely different.

We adopt the definition in [4] of analytic polyhedron. By an analytic polyhedron in *general position* we mean an analytic polyhedron as defined in [3, p. 288].

THEOREM 1. *Let $A \subset \mathbf{C}^n$ be the closure of a bounded analytic polyhedron in general position and let X be any open set in $\mathbf{C}^n$, containing A. Then the restriction map*

$$H^q(X, \mathcal{O}) \to H^q(X\backslash A, \mathcal{O}) \qquad (1 \leqq q \leqq n-2) \tag{1}$$

is bijective.

We proceed as in [2] except that now we take $G=B\backslash A$ where $B=\{z\in D;\ f_j(z)\in\Delta_j' \text{ for } j=1, \cdots, N\}$ where A is defined by $A=\{z\in D;\ f_j(z)\in\Delta_j \text{ for } j=1, \cdots, N\}$ where f_j are holomorphic in D, Δ_j' is some open neighborhood of $\bar{\Delta}_j$, and $\bar{B} \subset D$. (The argument in [2] can be simplified by dropping out the sets $U_{i_1}, \cdots, U_{i_q}$ which occur in the covering $X\backslash A$.) All we need to prove is the following lemma.

LEMMA. $H^p(G, \mathcal{O})=0$ *for* $1 \leqq p \leqq n-2$.

PROOF. For simplicity we take Δ_j to be the unit disc and Δ_j' to be a disc with radius $1+\epsilon$, homothetic to Δ_j. Clearly $G=\bigcup_{i=1}^N U_i$ where U_i is defined as B except for the additional condition $|f_i(z)|>1$. Thus, each U_i is also an analytic polyhedron. We next proceed analogously to [6, p. 349] and represent $f_{i_0\cdots i_p}$ in $U=\bigcap_{i=1}^N U_i$ as $\sum C_M(f_{i_0\cdots i_p})$ where $M=\{M', M''\}$ is a set of indices $j_1, \cdots, j_n$ such that the integration in $C_M(f)$ is taken over $|f_{j_1}|=\gamma_1, \cdots, |f_{j_n}|=\gamma_n$ where $\gamma_h=1$ if $j_h\in M''$ and $\gamma_h=1+\epsilon$ if $j_h\in M'$; the above integral representation is that given by the Cauchy-Weil formula [3],

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