

# PSEUDOCOMPACT ALGEBRAS, PROFINITE GROUPS AND CLASS FORMATIONS

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This note announces the main results obtained in a paper of the same title to appear in the Journal of Algebra, in which complete proofs can be found.

**Introduction.** We recall that a topological group  $G$  is a *profinite group* if it is the inverse limit of finite groups and that a  $G$ -module  $A$  is a *discrete  $G$ -module* if  $A = \bigcup A_H$ , where  $H$  runs through the open subgroups of  $G$  and  $A_H$  is the set of elements of  $A$  left fixed by  $H$  (cf. [4]). We note that if  $H$  is a normal subgroup of  $K$ , then  $A_H$  is a  $K/H$ -module. A *class formation* consists of a profinite group  $G$  and a  $G$ -module satisfying certain axioms which we do not repeat here: the reader will find them and their consequences in [1]. The reciprocity map for the formation gives a homomorphism

$$\omega_H: A_H \rightarrow H/H'$$

for each open subgroup  $H$  of  $G$  since  $H/H'$  is the group of the maximal abelian extension of  $H$  (cf. p. 179 of [6]). Let  $C_H$  be the kernel of  $\omega_H$  and let  $D_H$  be its cokernel. For each subgroup  $K$  of  $G$ , containing  $H$  as a normal subgroup, the exact sequence of  $K/H$ -modules

$$0 \rightarrow C_H \rightarrow A_H \rightarrow H/H' \rightarrow D_H \rightarrow 0$$

gives rise to homomorphisms

$$d_q: \hat{H}^{q-2}(K/H, D_H) \rightarrow \hat{H}^q(K/H, C_H)$$

as the composition of two coboundary maps.

**THEOREM 1.** *The following are equivalent for a class formation:*

- (i)  $\text{scd}_p G \leq 2$ ,
- (ii) *For every integer  $q$ , the map  $d_q$  induces an isomorphism onto on the  $p$ -primary components.*

The second condition is equivalent to a group theoretic property introduced by Kawada in [3].

For any field  $k$ , let  $G_k$  denote the Galois group of the separable closure of  $k$ . The following results about the associated class forma-

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