LEFT ALMOST PERIODICITY DOES NOT IMPLY RIGHT ALMOST PERIODICITY

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Let G be a topological group. A real valued continuous function f, defined on G, is left $\{$right$\}$ almost periodic iff for any $\epsilon>0$, there is a left $\{$right$\}$ syndetic subset[3] A of G such that $|f(ax)-f(x)|<\epsilon$ for all $a\in A$, $x\in G$. In this note, we shall show that a left almost periodic function is not necessarily right almost periodic even if the group G is a Lie group. This answers a problem in [3]. For the notions of almost periodic functions, we refer to [1], [4]. $C(X)$ denotes the set of all continuous real valued functions on the topological space X.

DEFINITION 1. Let N be a closed subgroup of a topological group G. We say that N splits in G if N is normal in G and there is a closed subgroup C such that

(i) $N\cap C=\{e\}$, e the identity of G.

(ii) $G=CN$.

(iii) The mapping $(c, n)\rightarrow cn$ is a homeomorphism of CXN onto G.

In this case G is said to be the semidirect product of N and C [5], [6].

PROPOSITION 1. *Assume G is a semidirect product of a compact normal subgroup N and a subgroup C. Let $f\in C(N)$. Define F on G by $F(cn)=f(n)$. Then $F\in C(G)$, and F is left almost periodic.*

PROOF. It is clear that F is well defined and belongs to $C(G)$. Let $\epsilon>0$, $|F(cn)-F(n)|=|f(n)-f(n)|=0<\epsilon$. Since $CN=G$, C is left syndetic. Thus F is left almost periodic. In fact, F is left periodic in the sense of [4].

PROPOSITION 2. *In addition to the assumption and notations of Proposition 1, we assume that there are elements $n\in N$, $n\neq e$, and a net $\{g_\nu\}$ in G such that $g_\nu n g_\nu^{-1}\rightarrow e$. If $f\in C(N)$, with $f(n)\neq f(e)$, then F is not right almost periodic.*

PROOF.

$$|F(g_\nu n g^{-1} g_\nu) - F(g_\nu)| = |F(g_\nu n) - F(g_\nu e)| = |f(n) - f(e)| = a \neq 0,$$

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[3] A subset S of G is left $\{$right$\}$ syndetic [4] iff there exists a compact subset K of G so that $G=SK$ $\{G=KS\}$.