ON NONLINEAR ELLIPTIC BOUNDARY-VALUE PROBLEMS

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The purpose of this note is to prove the solvability of a nonlinear elliptic equation with general boundary conditions. Nonlinear variational elliptic boundary-value problems have been considered by Browder in [4], [5] and by Visik.

In §1, we give the notations. In §2, we prove the solvability of the nonlinear elliptic equation with linear boundary conditions and in §3, we consider the case when we have a nonlinear boundary condition.

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1. Let G be a bounded, open subset of E^n with a C^∞ imbedding mapping of its boundary Γ into E^n. Let A be a linear elliptic differential operator of order $2m$ with coefficients defined on G; and $a(x, \xi)$ its characteristic form. Let $B_1, \cdots, B_m$ be m linear differential operators of orders r_j with coefficients defined on Γ and let $b_j(x, \xi)$ be their characteristic forms.

We set:

$$D_j = i^{-1}\partial/\partial x_j; \quad j = 1, \cdots, n,$$

$$D^\alpha = \prod_{j=1}^{n} D_j^{\alpha_j}; \qquad |\alpha| = \sum_{j=1}^{n} \alpha_j, \quad \alpha_j \geqq 0.$$

The elliptic boundary-problem $\{A; B_j; j=1, \cdots, m\}$ on G is assumed to be uniformly regular in the sense of Browder [3].

We now state our main assumption on $\{A; B_j\}$:

ASSUMPTION 1. *Let $\{A; B_j; j=1, \cdots, m\}$ be a uniformly regularly elliptic boundary problem on G. We assume that:*

(i) $a(x, \xi)/|a(x, \xi)| \neq -1$ *for x in G, ξ in R^n.*

(ii) *if* $c_{rj}(x, T, t) = \int_C \lambda^{r-1} b_j(x, \lambda N_x + T)[a(x, \lambda N_x + T) + t]^{-1} d\lambda$ *where C is a closed, Jordan rectifiable curve in the λ upper half plane containing all the m roots of $a(x, \lambda N_x + T) + t$ and N_x is the unit outer normal to Γ at x; T is any tangent vector to Γ at x; then there exists a positive constant c independent of x, t such that:*

$$|\operatorname{Det}(c_{rj}(x, T, t))| \geqq c \quad \textit{for } t \geqq t_0 > 0$$

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