GROTHENDIECK TOPOLOGIES OVER COMPLETE LOCAL RINGS

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1. **Introduction.** J. Tate [8] has introduced a theory of cohomological dimension for fields using the *étale* Grothendieck (=Galois) cohomology. In recent work, M. Artin has extended these methods to produce a dimension theory for noetherian preschemes. On the other hand, the author [5] has used the flat Grothendieck cohomology over a field to study certain duality questions (see also [7], [9] for the *étale* case); so it is natural to ask whether there exists a dimension theory based on the flat cohomology. We shall show that the answer is, in general, no. Full proofs will appear in [6].

2. **Terminology.** A Grothendieck topology is a pair consisting of a category Cat T and a set Cov T of families of morphisms of Cat T. They are subjected to the axioms:

(1) If ϕ is an isomorphism, $\{\phi\}\in$Cov T.

(2) If $\{U_i\to U\}\in$Cov T and $\{V_{ij}\to U_i\}\in$Cov T, for all i, then $\{V_{ij}\to U\}\in$Cov T.

(3) If $\{U_i\to U\}\in$Cov T and $V\to U$ is arbitrary, then $U_i\times_U V$ exists for each i, and $\{U_i\times_U V\to V\}\in$Cov T.

A presheaf (of abelian groups) on T is a contravariant functor from Cat T to the category of abelian groups, while a sheaf, F, is a presheaf which satisfies the axiom

(S) For all $\{U_i\to U\}\in$ Cov T, the natural sequence

$$F(U)\to\prod_i F(U_i)\rightrightarrows\prod_{i,j}F(U_i\times_U U_j)$$

is exact (i.e., $F(U)$ is mapped bijectively onto the set of all $x\in\prod_i F(U_i)$ whose images by the two maps shown agree in $\prod_{i,j}F(U_i\times_U U_j)$.) Roughly speaking, all that is done in Godement's book [2] for classical sheaf theory may be done in this general setting [1]. If X is a prescheme [3, Vol. I, p. 97], we let Cat T be the category of all preschemes Y which are separated, finitely presented, flat, and quasi-finite over X [3, Vol. I, p. 135, p. 144; Vol. IV, p. 5; Vol. II, p. 115]. Cov T consists of arbitrary families of flat morphisms whose disjoint sum is faithfully flat [3, Vol. IV, Part 2, p. 9]. It is known that these

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