ISOMORPHIC COMPLEXES. II

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In a preceding note [2] we showed that if K and L are n-complexes, then K and L are isomorphic iff the 1-sections of the first derived complexes of K and L are isomorphic. Since topological equivalence does not imply combinatorial equivalence for complexes this result fails to hold if the 1-sections are only required to be homeomorphic. However, for a large class of complexes we will show that this theorem is true under the weaker condition.

Throughout, s_p will denote a (rectilinear) p-simplex with vertices $a^0, a^1, \cdots, a^p$; K will denote a finite geometric simplicial complex with n-section K^n and first derived complex K'. For more details see [1, §1.2].

We first recall a definition and two theorems from [2].

DEFINITION 1. An n-complex K is *full* provided, for any subcomplex L of K which is isomorphic to s_p^1, $2 \leqq p \leqq n$, L^0 spans a p-simplex of K.

THEOREM 1. *If K and L are full n-complexes, then K and L are isomorphic iff K^1 and L^1 are isomorphic.*

THEOREM 2. *If K and L are n-complexes, then K and L are isomorphic iff $(K')^1$ and $(L')^1$ are isomorphic.*

DEFINITION 2. A complex K is said to be *taut* provided, K^1 has no vertex of order 2.

DEFINITION 3. A complex K is said to be *trim* if it is full and taut.

In each of the next three theorems we need only prove one implication for the equivalence since isomorphic complexes have homeomorphic realizations.

THEOREM 3. *If K and L are taut 1-complexes, then K and L are isomorphic iff $|K|$ and $|L|$ are homeomorphic.*

PROOF. Let $\phi: |K| \to |L|$ be a homeomorphism of $|K|$ onto $|L|$. If a is a vertex of K, then the order of $\phi(a)$ is not two since order is a topological property. So $\phi(a)$ is a vertex of L. Hence L has at least as many vertices as K. Similarly, using ϕ^{-1} instead of ϕ we obtain that K has at least as many vertices as L. So K and L have the same number of vertices. Therefore, $v: K \to L$ defined by

$$v(a) = \phi(a), \qquad a \in K^0$$