GENERATORS AND STRONG GENERATORS
IN ERGODIC THEORY

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The purpose of this note is to answer in the affirmative the question[1] raised by Rohlin [1], [2] as to whether every ergodic automorphism of a Lebesgue space with continuous measure possesses a countable generator. Rohlin has dealt with the case of automorphisms with finite entropy [2], and for this case he proved the existence of a countable generator with finite entropy. Automorphisms with infinite entropy such as the Bernoulli shift on a doubly infinite product of unit intervals were not amenable to Rohlin's method. It is well known that a generator of an automorphism with zero entropy is also a strong generator. Our result actually produces strong generators for arbitrary ergodic nonsingular one-one transformations, and thus, for the case of automorphisms with positive entropy these strong generators have infinite entropy.[2]

Rohlin imposed the weaker condition of aperiodicity rather than ergodicity and our result can be proved under this condition also, but in order to avoid undue emphasis on technicalities, we will assume that T is an ergodic nonsingular one-one transformation of a Lebesgue space $(X, \mathfrak{B}, m)$ with continuous measure such as the unit interval. Our result implies that every ergodic automorphism can be viewed as the shift associated with a stationary stochastic process (strong sense) with a countable number of states.

For the basic properties of measurable partitions and subalgebras cf. [1], [3].

A countable measurable partition ζ of X is called a generator [strong generator] if

$$\bigvee_{n=-\infty}^{\infty} T^n \zeta = \mathfrak{B} \left[\bigvee_{n=0}^{\infty} T^{-n} \zeta = \mathfrak{B} \right]$$