FUNCTIONAL DIFFERENTIAL EQUATIONS CLOSE TO DIFFERENTIAL EQUATIONS

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In a recent Research Problem [1], Bellman has raised the question of the behavior of solutions of the functional differential equation

$$u'(t) + au(t - r(t)) = 0 \tag{1}$$

when the lag function $r(t)$ is nearly constant for large t, and has also asked for conditions on the function r under which all solutions approach zero as $t \to \infty$. The purpose of this announcement is to initiate a study of various stability and oscillation problems for equations with perturbed lag functions, and to suggest that a modification of the familiar method of conversion to an integral equation can be profitably employed for these problems.

In this announcement, we characterize the asymptotic behavior of solutions of Equation (1) in case $r(t)$ tends to zero with a certain order as $t \to \infty$. We go beyond the problem posed by Bellman by establishing the asymptotic equivalence of Equation (1) and an approximating ordinary differential equation. In subsequent work, we shall deal with cases in which $r(t)$ is asymptotically constant, or is oscillatory, extensions to higher order equations, and so on. In particular, we shall deal with the remaining parts of Bellman's Research Problem. The first result obtained is as follows:

THEOREM 1. *Let $r(t)$ be continuous and nonnegative for $t \geqq t_0$ and assume that the following conditions are satisfied*:

$$\begin{aligned} &\text{(i)} && r(t) \to 0 \quad \textit{as} \quad t \to +\infty; \\ &\text{(ii)} && \int^{\infty} r(t)\,dt < \infty; \\ &\text{(iii)} && \inf_{t \geqq t_0} [t - r(t)] > -\infty. \end{aligned} \tag{2}$$

Then every continuous solution of Equation (1) *for $t \geqq t_0$ satisfies*

$$\lim_{t \to +\infty} u(t)e^{at} = c, \tag{3}$$

for some constant c. Moreover, for every c, $-\infty < c < +\infty$, there is a

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