UNIFORMLY BOUNDED REPRESENTATIONS OF THE UNIVERSAL COVERING GROUP OF SL(2, R)[1]

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Communicated by E. Hewitt, November 23, 1965

In a recent paper, Pukanszky [6] has classified all the irreducible unitary representations of the universal covering group G of $G_1 = \mathrm{SL}(2, \boldsymbol{R})$. We construct analytic continuations of the "principal series" of representations of G into certain domains in the complex plane. The representations obtained by continuation are uniformly bounded and, in special cases, they are equivalent to those constructed for G_1 by Kunze and Stein in [4]. In the course of the investigation, several interesting connections with special functions occur. Full details and proofs will be published elsewhere.

I. **The principal series and complementary series.** For the purposes of analytic continuation, we need realizations of the irreducible unitary representations of G which are somewhat different than those constructed in [6]. The group G may be parametrized as follows. $G = \{(\gamma, \omega) \mid \gamma \in \boldsymbol{C}, \ |\gamma| < 1; \ \omega \in \boldsymbol{R}\}$ (see [1, p. 594]). The principal series of irreducible unitary representations of G may be realized in $L_2(T)$, T the unit circle in $\boldsymbol{C}$. This was originally suggested by Bargmann [1, p. 616]. The representations are indexed by two parameters h and s, $h \in \boldsymbol{R}$, $-\frac{1}{2} < h \leqq \frac{1}{2}$, $s \in i\boldsymbol{R}$ (pure imaginary). We exclude the pair $h = \frac{1}{2}$, $s = 0$, which gives a reducible representation. For $f \in L_2(T)$, $g = (\gamma, \omega) \in G$, the representation operators are given by

$$[U_h(g, s)f](e^{i\theta}) \tag{1}$$
$$= e^{-2i\omega h}\left(\frac{1 + e^{i\theta}\bar{\gamma}}{1 + e^{-i\theta}\gamma}\right)^h \left| e^{i\theta}\bar{\gamma} + 1 \right|^{-1-2s}(1 + |\gamma|^2)^{1/2+s} f(e^{i\theta} \cdot g),$$

where $e^{i\theta} \cdot g = e^{2i\omega}[(e^{i\theta} + \gamma)/(e^{i\theta}\bar{\gamma} + 1)]$ is the natural action of G on T. We remark that, for any pair $(h, s) \in \boldsymbol{C}^2$, the operator defined by (1) is a bounded operator on $L_2(T)$ and the map $g \to U_h(g, s)$ is a continuous representation of G on $L_2(T)$. The principal series of irreducible unitary representations of G_1 is obtained from (1) by setting $h = 0$ and $h = \frac{1}{2}$.

For any fixed h, $-\frac{1}{2} < h \leqq \frac{1}{2}$, and $s \in i\boldsymbol{R}$, the representations

[1] The results presented in this paper are a part of the author's Ph.D. thesis at Brandeis University, written under the direction of Professor R. A. Kunze.

[2] Partially supported by the National Science Foundation, Contract NSF GP-89.