SPIN COBORDISM

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1. **Statements of results.** Ω_*^{Spin}, the Spin cobordism ring, has been studied by many, e.g. Wall [9, p. 294], Milnor [5] and [6], Novikov [7], and P. G. Anderson [3]. In this announcement we describe the additive structure of Ω_*^{Spin}, much of the multiplicative structure, characteristic numbers which determine Ω_*^{Spin}, and other properties.

We first state some technical results. Let $\mathfrak{A}$ denote the mod 2 Steenrod algebra, and let $Q_0 = Sq^1$ and $Q_1 = Sq^3 + Sq^2Sq^1$. If $a_1, a_2, \cdots, a_r \in \mathfrak{A}$, $\mathfrak{A}(a_1, a_2, \cdots, a_r)$ will denote the left ideal generated by $\{a_i\}$. All cohomology groups will have Z_2 coefficients unless otherwise stated. Let $p: BO\langle n\rangle \to BO$ be the fibre space such that $\pi_i(BO\langle n\rangle) = 0$ for $i < n$ and $p_*: \pi_i(BO\langle n\rangle) \approx \pi_i(BO)$ for $i \geqq n$. The following theorem is due to R. Stong [8].

THEOREM 1.1. *There is an element $\alpha_n \in H^n(BO\langle n\rangle)$ such that the map of $\mathfrak{A}$ into $\overline{H}^*(BO\langle n\rangle)$ given by $a \to a\alpha_n$ defines an isomorphism in dimensions less than $2n$ between $\mathfrak{A}/\mathfrak{A}(Sq^1, Sq^2)$ and $\overline{H}^*(BO\langle n\rangle)$ for $n \equiv 0$ (mod 8) and between $\mathfrak{A}/\mathfrak{A}(Sq^3)$ and $\overline{H}^*(BO\langle n\rangle)$ for $n \equiv 2$(mod 8).*

Let $\xi \in \overline{KO}^0(X)(X)$ be of filtration n [4], that is, ξ is trivial on the $n-1$ skeleton of X. Then there is a map $f_\xi: X \to BO\langle n\rangle$ such that pf_ξ is ξ. Let $[\xi] = \{f_\xi^*(\alpha_n)\} \subset H^n(X)$ for all f_ξ such that $pf_\xi = \xi$.

Let $J = (j_1, \cdots, j_k)$ be a sequence of integers with $j_i > 1$ and $k \geqq 0$. Let $P_J = P_{j_1} \cdots P_{j_k} \in H^{4n(J)}(B\,\mathrm{Spin})$, where $n(J) = \sum j_i$ and P_j is the jth Pontrjagin class. In [2], certain classes $\pi^i \in KO^0(BSO)$ were defined which behave very much like Pontrjagin classes. Under the map $B\,\mathrm{Spin} \to BSO$, π^i maps into a class which we also denote $\pi^i \in KO^0(B\,\mathrm{Spin})$. Let $\pi^J = \pi^{j_1} \cdots \pi^{j_k} \in KO^0(B\,\mathrm{Spin})$. Our main result from KO-theory is the following theorem.

THEOREM 1.2. *The filtration of π^J is $4n(J)$ if $n(J)$ is even, and is $4n(J)-2$ if $n(J)$ is odd. Furthermore, if $n(J)$ is even, there exists $X_J \in H^{4n(J)}(B\,\mathrm{Spin})$ such that $X_J \in [\pi^J]$ and $X_J \equiv P_J$ mod Im Q_0Q_1, and if $n(J)$ is odd, there exists $Y_J \in H^{4n(J)-2}(B\,\mathrm{Spin})$ such that $Y_J \in [\pi^J]$ and $Sq^2 Y_J \equiv P_J$ mod Im Q_0Q_1.*

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