MANIFOLDS WITH $\pi_1 = Z$

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In this note we announce some results extending results of S. P. Novikov ([6] and [7]), the author [2], and C. T. C. Wall [8]. In the above papers it is shown how to characterize the homotopy type of 1-connected smooth closed manifolds of dimension $n \geqq 5$, $n \not\equiv 2 \bmod 4$, and how to reduce the diffeomorphy classification of such manifolds to homotopy theory, with similar results for bounded manifolds in [8]. We show how to adapt these techniques to manifolds with $\pi_1 = Z$ and get analogous results. By studying the "mapping torus" using these results one may obtain results on existence and pseudo-isotopy of diffeomorphisms, (see [3]).

One has for example the situation of a closed smooth manifold M^n and a map $f: M^n \to X$, such that the normal bundle ν of M in S^{n+k} is induced by f from a bundle ξ over X. One does surgery on M with respect to the map f, i.e. if W is the cobordism determined by the surgery, then f extends to a map $F: W \to X$ such that the normal bundle of W in $S^{n+k} \times I$ is induced from ξ by F. In case M is simply connected many conditions facilitate the surgery, such as the Whitney embedding theorem, and the Hurewicz theorem, so that, with appropriate hypothesis on X and ξ, it is often possible to do surgery to create a manifold homotopy equivalent to X. The case of a non-zero fundamental group poses many problems, but if $\pi_1 M = Z$, one can reduce the situation to the simply connected case by using extra geometrical structure. The idea is to consider a 1-connected manifold U^n with two 1-connected boundary components, $\partial U = A_0 \cup A_1$, with $f: A_0 \to A_1$ a diffeomorphism, and consider the identification space M^n of U with $a \in A_0$ identified to $f(a) \in A_1$. Then M^n is closed and connected with $\pi_1 M = Z$, and it can be shown using surgery that any smooth connected M^n with $\pi_1 M = Z$, $n \geqq 5$ can be represented this way. One may then study U and A_0, A_1 using the techniques of surgery on 1-connected manifolds and then use this to obtain information about M.

In §1 we deal with closed manifolds and in §2, with manifolds with boundary. In §2 we examine in particular the case of homology circles, which gives certain results on the complements of higher dimensional knots (e.g. Corollaries 2.3 and 2.4).

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