MULTILINEAR LEBESGUE-BOCHNER-STIELTJES INTEGRAL

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Communicated by A. Zygmund, November 22, 1965

In this paper we introduce an integral of the form $\int u(f_{ji}, d\mu_j)$ where u is a multilinear operator from the product of the Banach spaces Y_{ji}, Z_j $(j=1, \cdots, m,\ i=1, \cdots, k_j)$ into a Banach space W, and f_{ji} are Lebesgue-Bochner summable functions, and μ_j are vector volumes.

The above integral is a generalization of the integral $\int u(f, d\mu)$ developed in [**1**]. An integral similar to the last integral, developed in a different way, one can find in Bourbaki [**10**, Chapter V, p. 48–49]. For applications, see the following paper in this volume.

1. **Properties of vector volumes.** Let R be the space of reals and Y_i, Z_i, W be seminormed spaces. Denote by $L(Y_1, \cdots, Y_k; W)$ the space of all k-linear continuous operators u from the space $Y_1 \times \cdots \times Y_k$ into the space W. The norms of elements in the above spaces will be denoted by $| \ |$.

The family of sets V of an abstract space X will be called a prering if for any two sets $A_1, A_2 \in V$ we have $A_1 \cap A_2 \in V$ and there exists disjoint sets $B_1, \cdots, B_k \in V$ such that $A_1 \backslash A_2 = B_1 \cup \cdots \cup B_k$.

A nonnegative function v on a prering V is called a positive volume or when there is no confusion just volume if it is countably additive, that is for every countable family of disjoint sets $A_t \in V$ $(t \in T)$ such that $A = \bigcup_T A_t \in V$ we have $v(A) = \sum_T v(A_t)$.

A function μ from a prering V into a Banach space Z is called a vector volume or simply volume when there is no confusion possible if the function μ is finite additive on V and for some positive volume v we have

$$|\mu(A)| \leqq v(A) \quad \text{for all } A \in V.$$

It follows from this definition and from the definition of a prering that every volume is countably additive.

THEOREM 1. *Let V_i be a prering of sets of a space X_i $(i=1, \cdots, k)$. Denote by $V = V_1 \times \cdots \times V_k$ the family of all sets of the form $A = A_1 \times \cdots \times A_k$ where $A_i \in V_i$. Then V is a prering of sets of the space $X = X_1 \times \cdots \times X_k$.*

[1] This work was partially supported by NSF grant GP-2565.