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CASE INSTITUTE OF TECHNOLOGY

## ON THE EQUATION $f^n + g^n = 1$

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There is a close relationship between Fermat's last theorem and the family of solutions  $f$  and  $g$  of the functional equation

$$(1) \quad x^n + y^n = 1.$$

If, for example,  $S_D$  denotes the class of all pairs  $(f, g)$  of single valued functions  $f$  and  $g$  meromorphic in a domain  $D$  and having the additional property that, for some  $z_0$  in  $D$ ,  $f(z_0)$  and  $g(z_0)$  are both positive rationals, then either, for  $n > 2$ , (1) has no solutions in  $S_D$  or Fermat's last theorem is not true.

In this note we discuss the solutions of (1) meromorphic in the complex plane. We shall call such solutions  $M_c$  solutions.

**THEOREM 1.** *For  $n = 2$ , all  $M_c$  solutions are of the form*

$$(2) \quad f = \frac{2\beta(z)}{1 + \beta(z)^2} \quad \text{and} \quad g(z) = \frac{1 - \beta(z)^2}{1 + \beta(z)^2}.$$

**PROOF.** This follows directly from a theorem on uniformization [1]. We need only note that for  $n = 2$ , (1) is of genus zero and that the rational solution (2), with  $\beta(z) = z$ , maps the whole  $z$ -plane in a 1-1 manner on the Riemann surface of (1).

**THEOREM 2.** *For  $n = 3$ ,  $M_c$  solutions exist. One such solution is given by:*

$$(3) \quad \begin{aligned} f &= 4^{-1/6}(\wp')^{-1}(1 + 3^{-1/2} \cdot 4^{1/3} \wp), \\ g &= 4^{-1/6}(\wp)^{-1}(1 - 3^{-1/2} \cdot 4^{1/3} \wp), \end{aligned}$$

where  $\wp$  is a Weierstrass  $\wp$ -function.