THE STRICT TOPOLOGY AND COMPACTNESS IN THE SPACE OF MEASURES

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The strict topology β on the space $C(S)$ of bounded complex valued continuous functions on a locally compact space S was introduced by R. C. Buck [1] and has been studied by Glicksberg [4] and Wells [9]. Among the problems in mathematics which have seen successful applications of the strict topology are various ones in spectral synthesis (Herz [6]) and spaces of bounded analytic functions (Shields and Rubel [8]). In spite of these successes there has as yet been no detailed investigation of the relationship between this topological vector space and its adjoint. This is an announcement of some results from an attempt at such an investigation. The complete proofs, as well as those of some additional results and extensions of the present theorems, will appear elsewhere.

In particular we are interested in a question posed by Buck. Is $C(S)$ with the strict topology a Mackey space? As yet no characterization of those spaces S for which the answer is affirmative is available. However, we can prove a much stronger result whenever S is paracompact—a class which includes all σ-compact spaces and topological groups.

In the following $C_0(S)$ will denote the subspace of $C(S)$ consisting of all functions which vanish at infinity, and $C_c(S)$ those which vanish off some compact set. If $\phi \in C(S)$ then let $N(\phi)=\{s:\phi(s)\neq 0\}$, spt $(\phi)=N(\phi)^-$ (the closure of $N(\phi)$), $\|\phi\|_\infty=\sup\{|\phi(s)|:s\in S\}$, and $V_\phi=\{f\in C(S):\|\phi f\|_\infty\leqq 1\}$.

The *strict topology* β on $C(S)$ is defined by the neighborhood basis at the origin consisting of the sets $\{V_\phi:\phi\in C_0(S)\}$. Some of Buck's results are that $C(S)_\beta$ is complete, the β-bounded and norm bounded subsets of $C(S)$ are the same, $C_c(S)$ is β-dense in $C(S)$, and the adjoint of $C(S)_\beta$ is $M(S)$, the space of bounded regular Borel measures on S.

We will denote by "β-weak *" the weak star topology on $M(S)$

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